Suan Sunandha Rajabhat University
Faculty of Education, Branch of Mathematics
Final Examination, Semester 1/2016

Subject ID	Course Name	Test Time	Full Scores
MAT2303	Abstract Albegra	9am - 11am Tue 13 Dec 2016	105 points 35%

Name.. **ID**.................................. **Section**..............

Direction

1. 10 questions and 10 pages.
2. Write obviously your name, id and section all pages.
3. Without calculators and communication tools.
4. Don't take text books and others come to thetest room.
5. Cannot answer sheets out of test room.
6. Deliver to the staff if you make a mistake in the test room.

Signature

....

Lecturer: Thanatyod Jampawai, Ph.D.

No.	1	2	3	4	5	6	7	8	9	10	Total
Score											

1. **(12 points)** Write answers in the right blanks

1.1 Find all **prime elements** of $\mathbb{Z}_8$. ________________

1.2 Compute **number of all zero divisors** of $\mathbb{Z}_{100}$. ________________

1.3 Find the **remainder** of division 7^{50} by 13. ________________

1.4 Find number of polynomials of degree 2 of $\mathbb{Z}_5[x]$. ________________

1.5 In $\mathbb{Z}[x]$, find $\gcd(x^3 - 1, x^{12} - 1)$. ________________

1.6 If $x^2 + a \in \mathbb{Z}_3[x]$ is irreducible, find a. ________________

1.7 Compute number of all possible rational roots of $12x^3 + 3x^2 - 7x - 6$. ________________

1.8 If the remainder of division $x^3 + ax^2 - x + 1$ by $x - 1$ is 3, what is a ? ________________

1.9 If $ax^3 + 5x^2 + bx - 1 \in \mathbb{Z}[x]$ is a reciprocal polynomial, find $a + b$. ________________

1.10 Let $\dfrac{1}{x^2 - 3x + 2} = \dfrac{A}{x - 2} + \dfrac{B}{x - 1}$. What is $A + B$? ________________

1.11 Find condition of a which $x^{15} - 3x^2 + 6x - a$ is irreducible in $\mathbb{Z}[x]$. ________________

1.12 Give an example of irreducible polynomial(s) of degree 10 in $\mathbb{Q}[x]$. ________________

2. Verify your answers

   2.1 **(5 points)** Let the upper triangular matrix set be

$$U = \left\{ \begin{bmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{bmatrix} : a, b, c, d, e, f \in \mathbb{Z} \right\}.$$

   Determine whether U is a subring of ring $M_{33}(\mathbb{Z})$.
   (Prove by theorem, S is a subring of R if $\forall x, y \in S, x - y \in S$ and $xy \in S$)

   2.2 **(4 points)** Let R be a ring. Prove that

$$\text{if } I \text{ and } J \text{ are ideals of } R, \quad \text{then} \quad I \cup J \quad \text{is an ideal of } R.$$

3. Verify your answers

   3.1 **(5 points)** Determine whether I is an ideal of $M_{33}(\mathbb{R})$. If

$$I = \left\{ \begin{bmatrix} x & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} : x \in \mathbb{R} \right\}.$$

   3.2 **(6 points)** Let $\varphi : \mathbb{Z} \to \mathbb{Z}_2$ defined by $\varphi(x) = \bar{x}^2$.

   (a) Is φ a ring homomorphism ? Verfy your answer.

   (b) Find $Ker(\varphi)$

4. Verify your answer.

   4.1 **(5 points)** Find all prime ideals and maximal ideals of $\mathbb{Z}_{15}$

   4.2 **(5 points)** Fill blanks in the table and find all prime elements and irreducible elements of $\mathbb{Z}_{12}$.

$\bullet$	0	1	2	3	4	5	6	7	8	9	10	11
0												
1												
2												
3												
4												
5												
6												
7												
8												
9												
10												
11												

5. Verify your answer.

   5.1 **(6 points)** In quotient ring $\mathbb{Z}_{12}/(6)$

      a. List all elements of $\mathbb{Z}_{12}/(6)$

      b. Write out all inverses with multiplication of elements in $\mathbb{Z}_{12}/(6)$

   5.2 **(6 points)** Let $(a, b), (c, d) \in \mathbb{Z}_4 \times \mathbb{Z}_6$. Define

$$(a, b) \cdot (c, d) = (ac, bd).$$

      a. List all elements of $\mathbb{Z}_4 \times \mathbb{Z}_6$

      b. Find all zero divisors of elements in $\mathbb{Z}_4 \times \mathbb{Z}_6$

6. Explain your answers

   6.1 **(6 points)** In polynomials ring $\mathbb{Z}_4[x]$, find all possible $a, b, c \in \mathbb{Z}_4$ whose

$$(ax^2 + bx + c)^2 = 1$$

   6.2 **(6 points)** Show that $x^2 + x + 1$ is irreducible in $\mathbb{Z}_6[x]$ by **contradiction**.

7. **(10 points)** Explain your answers

   a. Is $x^2 + x + 1$ irreducible in $\mathbb{Z}_2[x]$? Why?

   b. Is $\mathbb{Z}_2[x]/(x^2 + x + 1)$ a field ? Why?

   c. List all elements of $\mathbb{Z}_2[x]/(x^2 + x + 1)$.

   d. Find all inverses of elements in $\mathbb{Z}_2[x]/(x^2 + x + 1)$.

8. Explain your answers

   8.1 **(5 points)** Let $p(x) = x(x+1)(x+2)(x+3)+1 \in \mathbb{Z}[x]$ (Show that it can be factors).

      a. Prove that $p(x)$ is reducible in $\mathbb{Z}[x]$ (Show that it can be factors).

      b. Find all roots of $p(x)$.

   8.2 **(5 points)** Let α, β and γ be a roots over $\mathbb{Z}$ of

$$x^3 - x^2 + 2x + 1.$$

   Compute $(\alpha+\beta)(\alpha+\gamma)(\beta+\gamma)$.

9. In $\mathbb{R}[x]$,

   9.1 **(5 points)** Find all roots in $\mathbb{C}$ of

$$(x^2 - x - 6)^3 + (x^2 - 3x - 10)^3 = (2x^2 - 4x - 16)^3.$$

   9.2 **(6 points)** Find all roots in $\mathbb{C}$ of

$$x^4 + 2x^3 - 13x^2 + 2x + 1 = 0.$$

ID.. **Section**....................

10. Write your answers (Write in Thai or English).

    10.1 **(4 points)** List all **technical terms** of Abstract Algebra (MAT2303) in your remember and give thier meanings or definitions.

    10.2 **(4 points)** Describe **your feeling** about Abstract Algebra (MAT2303) to tell your juniors.