



Suan Sunandha Rajabhat University
Faculty of Education, Branch of Mathematics
Final Examination, Semester 1/2017

ID Subject MAT2303	Course Name Abstract Algebra	Test Time 1pm - 4pm Fri 8 Dec 2017	Full Scores 105 points 30%
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Name..... ID..... Section.....

Direction

1. 10 questions and 11 pages.
2. Write obviously your name, id and section all pages.
3. Can use a calculator(s) but can not use communication tools.
4. Don't take text books and others come to the test room.
5. Cannot answer sheets out of test room.
6. Deliver to the staff if you make a mistake in the test room.

Signature

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Teacher: Thanatyod Jampawai, Ph.D.

No.	1	2	3	4	5	6	7	8	9	10	Total
Scores											

1. (10 points) Write answers in the right blanks

1.1 What is the **inverse** of $(5) + 2$ in quotient ring $\mathbb{Z}_{25}/(5)$? _____

1.2 Find $\text{Ker}(\varphi)$ if $\varphi : \mathbb{Z} \rightarrow \mathbb{Z}_5$ defined by $\varphi(x) = (\bar{x})^5$ _____

1.3 Compute the **number of all zero divisors** of \mathbb{Z}_{23017} _____

1.4 Find **all prime ideals** of \mathbb{Z}_{625} _____

1.5 Find **all irreducible elements** in \mathbb{Z}_9 _____

1.6 Give one example of a **field of order 121** _____

1.7 What is the **inverse** of $(x^2 + 1) + x$ in $\mathbb{Z}_3/(x^2 + 1)$ _____

1.8 Give an **irreducible polynomial** of degree 5 in $\mathbb{Q}[x]$ _____

1.9 Find **all possible rational roots** of $6x^3 - x^2 + x - 2$ _____

1.10 If 5 is a root of $x^5 - 5x^4 - 5x^3 + a$, what is a _____

2. Explain your answers and verify.

2.1 (5 points) Let $a, b \in \mathbb{Z}$. Define

$$a \oplus b = a + b$$

$$a \odot b = 2ab$$

Determine whether $(\mathbb{Z}, \oplus, \odot)$ is a ring

2.2 (5 points) Let $I = \left\{ \begin{bmatrix} 0 & 0 & 0 \\ 0 & x & 0 \\ 0 & 0 & 0 \end{bmatrix} : x \in \mathbb{R} \right\}$. Determine whether I is an **right ideal**, **left ideal** or **ideal** of $M_{33}(\mathbb{R})$.

3. Explain your answers and verify.

3.1 **(6 points)** Let $\varphi : \mathbb{Z} \rightarrow \mathbb{Z}_{20}$. Define $\varphi(x) = 16\bar{x}$.

- (a) **(2 points)** Show that φ is a ring homomorphism
- (b) **(2 points)** Find $\text{Ker}(\varphi)$ and $\text{Im}(\varphi)$
- (c) **(2 points)** Describe $\mathbb{Z}/5\mathbb{Z} \cong (4)$

3.2 **(4 points)** In a quotient ring $\mathbb{Z}_{20}/(10)$

- (a) **(2 points)** Write out element in $\mathbb{Z}_{20}/(10)$
- (b) **(2 points)** Find all inverses of elements in $\mathbb{Z}_{20}/(10)$

4. Explain your answers and verify.

4.1 (5 points) Give **two zero divisors** in $M_{22}(\mathbb{R})$ and verify your answers.

4.2 (5 points) Find all maximal ideals and prime ideals in \mathbb{Z}_{65670}

5. Explain your answers and verify.

5.1 **(5 points)** What is remainder when 2017^{2560} is divided by 13 (Use the little fermat's theorem)

5.2 **(5 points)** In ring $\mathbb{Z}[\sqrt{-7}]$

(a) **(2 points)** Find all units in $\mathbb{Z}[\sqrt{-7}]$

(b) **(3 points)** Give example(s) of element(s) in $\mathbb{Z}[\sqrt{-7}]$ to show that it is not U.F.D.

6. Explain your answers and verify.

6.1 (5 points) Find all irreducible elements in \mathbb{Z}_{22}

6.2 (5 points) Find all prime elements in \mathbb{Z}_{12}

7. (10 points) In polynomial ring $\mathbb{Z}_3[x]$

7.1 (2 points) Show that $2x^2 + 2$ is irreducible in $\mathbb{Z}_3[x]$

7.2 (3 points) Write out elements in a field $\mathbb{Z}_3[x]/(2x^2 + 2)$

7.3 (5 points) Find all inverses of elements in $\mathbb{Z}_3[x]/(2x^2 + 2)$

8. Explain your answers and verify.

8.1 (5 points) What is the G.C.D. of polynomial

$$x^4 + x^3 - x^2 + x - 2 \quad \text{and} \quad x^5 + x^4 + x^3 + x^2 - 2x - 2 \quad \text{in } \mathbb{Z}[x]$$

8.2 (5 points) Show that $x^4 + x^3 + x^2 + x + 1$ is irreducible in $\mathbb{Z}[x]$

9. Explain your answers and verify.

9.1 **(5 points)** Let $p(x), q(x) \in \mathbb{Z}_3[x]$. If $p(x) = (q(x))^2$ and $p(x) = x^4 + 4x^3 + 2x + 4$. Find $q(x)$

9.2 **(5 points)** Find all roots in \mathbb{C} of

$$(x - 101)^4 + (x - 99)^4 = 82$$

10. Explain your answers

10.1 (5 points) Find all roots in \mathbb{C} of

$$x^5 + 3x^4 + 5x^3 + 5x^2 + 3x + 1$$

10.2 (5 points) What did you learn from this class.