Suan Sunandha Rajabhat University
Faculty of Education, Branch of Mathematics
Final Examination, Semester 2/2016

ID Subject	**Course Name**	**Test Time**	**Full Scores**
MAT12305	Linear Algebra	9am - 12am Thur 4 May 2017	105 points 30%

Name.. **ID**.................................... **Section**...............

Direction

1. 10 questions and 12 pages.
2. Write obviously your name, id and section all pages.
3. Can use a calculator(s) but can not use communication tools.
4. Don't take text books and others come to the test room.
5. Cannot answer sheets out of test room.
6. Deliver to the staff if you make a mistake in the test room.

Signature

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Lecturer: Thanatyod Jampawai, Ph.D.

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	

1. **(10 points)** Write answers in the right blanks

1.1 Let $T(x, y) = (x + y, x - y)$. Find $T^{-1}(4, 2)$ ________________

1.2 Let $T_1 \circ T_2^{-1}(x, y) = (2x, 3y)$ If $T_2(1, 1) = (3, 4)$, find $T_1^{-1}(6, 12)$ ________________

1.3 Compute the **rank** of $T(x, y, z) = (x + 2y + z, y + z, x + y)$ ________________

1.4 In $\mathbb{P}_3$, compute the norm $\|1 + 2x - 2x^2\|$ ________________

1.5 If $\{(a, 1, b), (1, 2, -2), (-2, 2, 1)\}$ is an orthogonal basis, what is $a + b$. ________________

1.6 Let $\boldsymbol{u}$ and $\boldsymbol{v}$ be vectors in an inner product space. If $\boldsymbol{u}$ is orthogonal to $\boldsymbol{v}$ and $\|\boldsymbol{u} - \boldsymbol{v}\| = 5$, compute $\|\boldsymbol{u} - \boldsymbol{v}\|$ ________________

1.7 Let $W = \{(1, 1, 1), (1, 0, 1)\}$. Find $W^{\perp}$ (spanning) ________________

1.8 Compute all eigenvalues of $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 1 & 1 & 3 \end{bmatrix}$ ________________

1.9 Let $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$. Compute A^{2017}. ________________

1.10 Compute all eigenvalues of $\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}^{10}$. ________________

2. **(10 points)** Explain your answers

2.1 **(5 points)** Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by

$$T(x, y, z) = (x + 2y, -y + z, x + y + z)$$

Show that T is a linear operator (linear transformation).

2.2 **(5 points)** Find Nullity(T_A) and Rank(T_A) where

$$A = \begin{bmatrix} 1 & -1 & 0 & 3 \\ 0 & 1 & -1 & 0 \\ -2 & -3 & 0 & -1 \\ 0 & -1 & 0 & 1 \\ 0 & 1 & 0 & -1 \end{bmatrix}$$

3. **(10 points)** Let T_1 and T_2 be linear operators on $\mathbb{R}^3$ such that

$$T_1(x, y, z) = (x + y, x + z, y + z)$$
$$T_2 \circ T_1(x, y, z) = (x + 2y - z, 2x, x + 2y + z)$$

Find $T_2(x, y, z)$ and $T_1 \circ T_2(x, y, z)$

4. **(10 points)** Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation defined by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} 2x_1 - x_2 \\ x_1 + x_2 + x_3 \\ x_2 - x_3 \end{bmatrix}$$

Find $[T]_B$ where $B = \left\{ \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} \right\}$ is a basis for $\mathbb{R}^3$

5. **(10 points)** Explain your answers

   5.1 **(5 points)** Let $\boldsymbol{u} = (a, b)$ and $\boldsymbol{v} = (c, d)$. Defined an inner product on $\mathbb{R}^2$ by

   $$\langle \boldsymbol{u}, \boldsymbol{v} \rangle = 4ac + 5bd$$

   Find **cosine** of the angle between $(1, 2)$ and $(2, -1)$

   5.2 **(5 points)** Let $A = \begin{bmatrix} 1 & 1 \\ a & 0 \end{bmatrix}$ be **orthogonal** to $B = \begin{bmatrix} b & 2 \\ 1 & 2 \end{bmatrix}$ in $M_{22}(\mathbb{R})$.
   If $\|A\| = 2$, what is the **norm** of B.

6. **(10 points)** Explain your answers

6.1 **(7 points)** Let $\boldsymbol{u}, \boldsymbol{v}$ and $\boldsymbol{w}$ be vectors in an inner product space. Suppose that

$$\boldsymbol{u} + \boldsymbol{v} + \boldsymbol{w} = \boldsymbol{0}, \quad \|\boldsymbol{u} + \boldsymbol{v}\| = 5, \quad \|\boldsymbol{v} + \boldsymbol{w}\| = 3 \quad \text{and} \quad \|\boldsymbol{v}\| = \sqrt{10}$$

Compute $\langle \boldsymbol{u}, \boldsymbol{w} \rangle$

6.2 **(3 points)** Let $B = \{(1, 2, -1), (2, 1, 4), (5, -10, 0)\}$ be an **orthogonal basis** for $\mathbb{R}^3$. Evaluate $[\boldsymbol{v}]_B$ where $\boldsymbol{v} = (3, 2, -1)$

7. **(10 points)** Use Gram-Schmidt process to transform the basis B into **orthonormal basis**.

$$B = \{(-1, 0, 1, 0), (0, -1, 0, 1), (1, 1, 1, 0), (0, 1, 1, 1)\}$$

8. **(10 points)** Explain your answers

8.1 **(5 points)** Let A be in $M_{22}(\mathbb{R})$ that has 3 and -1 to be eigenvalues corresponding to eigenvectors $\begin{bmatrix}1\\2\end{bmatrix}$ and $\begin{bmatrix}0\\1\end{bmatrix}$, respectively. What is the matrix A ?

8.2 **(5 points)** If 3 is an eigenvalue of

$$A = \begin{bmatrix} a & 2 & 3 \\ -1 & 1 & -3 \\ 2 & 4 & 9 \end{bmatrix}$$

Compute other eigenvalues.

9. **(20 points)** Use diagonalization to find A^{10} where

$$A = \begin{bmatrix} 2 & 3 & -2 \\ 2 & 3 & 0 \\ 6 & -6 & 7 \end{bmatrix}$$

10. **(5 points)** Review Linear Algebra MAT2305 (Don't over claim).