Suan Sunandha Rajabhat University
Faculty of Education, Division of Mathematics
Final Examination, Semester 2/2017

ID Subject	Course Name	Test Time	Full Scores
MAT2305	Linear Algebra	1pm - 4pm Mon 23 Apr 2018	105 points 30%

Name.. **ID**.................................... **Section**...............

Direction

1. 10 questions and 12 pages.
2. Write obviously your name, id and section all pages.
3. Can use a calculator(s) but can not use communication tools.
4. Don't take text books and others come to the test room.
5. Cannot answer sheets out of test room.
6. Deliver to the staff if you make a mistake in the test room.

Signature

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Lecturer: Thanatyod Jampawai, Ph.D.

	1	2	3	4	5	6	7	8	9	10	

1. **(10 points)** Write answers in the right blanks

1.1 Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation. If $T(1,2) = (3,4)$, find $T(4,8)$. ______________

1.2 If $\mathbb{R}^{15} \to \mathbb{R}^{10}$ be a linear transformation and $\text{rank}(T) = 6$. Find $\text{nullity}(T)$. ______________

1.3 Let $T(x,y) = (ax, bx + cy)$. If $T(2x, 3y) = (4x, 6x + 9y)$, compute $a + b + c$. ______________

1.4 Assume that $T : \mathbb{R}^3 \to \mathbb{R}^2$ is a linear transformation and $\text{Ker}(T) = \{\mathbf{0}\}$.
If $T^{-1}(1,2) = (3,1,2)$ and $T^{-1}(2,1) = (1,2,3)$, compute $T(4,3,5)$. ______________

1.5 Let $\boldsymbol{u}$ and $\boldsymbol{v}$ be vectors in an inner product space. If $\|\boldsymbol{u}\| = \|\boldsymbol{v}\| = 3$ and $\langle \boldsymbol{u}, \boldsymbol{v} \rangle = 1$, compute $\|\boldsymbol{u} - \boldsymbol{v}\|$. ______________

1.6 Let $A = \begin{bmatrix} \sin\theta & \cos\theta \\ 2 & 2 \end{bmatrix}$ where $\theta \in \mathbb{R}$. Find $\|A\|$ (standard norm in $M_{22}(\mathbb{R})$). ______________

1.7 Let $f(x) = x^2$. Compute $\|f\|$ in $C[0,5]$. ______________

1.8 Let $\mathcal{B} = \{(1,3), (-3,1)\}$ be a basis for $\mathbb{R}^2$. What is $[(7,1)]_{\mathcal{B}}$. ______________

1.9 If λ_1 and λ_2 are two eigenvalues of T where $T(x,y) = (-2x - y, 4x + 3y)$, compute $\lambda_1 + \lambda_2$. ______________

1.10 Find all eigenvalues of $\begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}^4$. ______________

2. Explain your answers

   2.1 **(4 points)** Let $T : M_{22}(\mathbb{R}) \to \mathbb{R}^3$ defined by

$$T\left(\begin{bmatrix} x & y \\ z & w \end{bmatrix}\right) = (x+y, y+z, z+w).$$

   Show that T is a linear transformation

   2.2 **(4 points)** Assume that T is a linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^3$. If

$$T\left(\begin{bmatrix} 3 \\ 4 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \quad \text{and} \quad T\left(\begin{bmatrix} 4 \\ 5 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}.$$

   What is formula of $T(x, y)$.

3. Let T be a linear operator on $\mathbb{R}^3$ defined by

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x+y+z \\ x-y+z \\ 3x+y+3z \end{bmatrix}$$

   3.1 **(4 points)** Find Ker(T) and nullity(T)

   3.2 **(4 points)** Find Ran(T) and rank(T)

4. Let T_1 and T_2 be linear operator on $\mathbb{R}^3$. If

$$T_2(x,y,z) = (x, x+y, x+z) \quad \text{and} \quad (T_1 \circ T_2)^{-1}(x,y,z) = (x+y, x+z, y+z)$$

   4.1 **(4 points)** Find $(T_1 \circ T_2)(x,y,z)$

   4.2 **(4 points)** Find $T_1(x,y,x)$

5. **(10 points)** Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation defined by

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x+y-z \\ x-y+z \\ x+y+z \end{bmatrix}.$$

If $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 6 \\ 13 \end{bmatrix}, \begin{bmatrix} 1 \\ 5 \\ 10 \end{bmatrix}, \begin{bmatrix} 1 \\ 4 \\ 8 \end{bmatrix} \right\}$ is a basis for $\mathbb{R}^3$. Compute $[T]_{\mathcal{B}}$

6. Explain your answers

6.1 **(5 points)** Let $f, g \in C[a, b]$ and $a < b$. An inner product on $C[a, b]$ is defined by

$$\langle f, g\rangle = \frac{1}{b-a}\int_a^b f(x)g(x)\,dx$$

If $\|f\|^2 = 0.5$ where $f(x) = \sin x$. Compute the **minimum** of $b - a$. (Hint: $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$)

6.2 **(6 points)** Let $\boldsymbol{u}$ and $\boldsymbol{v}$ be vectors in an inner product space.
If $\|\boldsymbol{u}\| = 1$, $\|\boldsymbol{v}\| = 4$ the angle between $\boldsymbol{u}$ and $\boldsymbol{v}$ is $60°$. Compute $\dfrac{\|\boldsymbol{u}+\boldsymbol{v}\|}{\|2\boldsymbol{u}+\boldsymbol{v}\|}$

7. Let $\mathcal{B} = \{(1,2,3),(a,b,11),(-5,1,1)\}$ be an **orthogonal basis** for $\mathbb{R}^3$.

   7.1 **(5 points)** Find a and b

   7.2 **(5 points)** Compute $[(2,5,6)]_{\mathcal{B}}$

8. **(10 points)** Use Gram-Schmitd process to find **orthonormal basis** of

$$\mathcal{B} = \{(1,1,1),(1,0,1),(1,1,0)\}$$

9. **(20 points)** Use diagonalization to compute A^{10} where

$$A = \begin{bmatrix} 0 & 1 & 2 \\ -4 & 1 & 4 \\ -5 & 1 & 7 \end{bmatrix} \quad \text{(there is one eigenvalue to be 1)}$$

10. Explain your answers

    10.1 **(5 points)** Let $T(x, y) = (-x + 4y, 3x + 3y)$. Find eigenvalues and eigenspaces of T.

    10.2 **(5 points)** Choose some topic(s) in linear algebra (MAT2305) to teach your student and write **lesson plan** (sketch).