



Suan Sunandha Rajabhat University
Faculty of Education, Division of Mathematics
Midterm Examination, Semester 2/2019

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| ID Subject MAP2406 | Course Name Mathematical Analysis | Test Time 5pm - 8pm Thur 5 Mar 2020 | Full Scores 105 points 30% |
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Name..... ID..... Section.....

Direction

1. 10 questions and one Extra of all 12 pages.
2. Write obviously your name, id and section all pages.
3. Don't take text books and others come to the test room.
4. Cannot answer sheets out of test room.
5. Deliver to the staff if you make a mistake in the test room.

Your signature

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Lecturer: Assistant Professor Thanatyod Jampawai, Ph.D.

| No. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | Extra |
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| Scores | | | | | | | | | | | |

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| Total |
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1. (10 points) Let a, b and c be real numbers. Prove that

$$a^2 + b^2 + c^2 \geq ab + ac + bc.$$

2. (10 points) Let x and y be real numbers. Prove that

$$|x + y| = |x| + |y| \quad \text{if and only if} \quad xy \geq 0.$$

3. (10 points) Define the set

$$A = \left\{ \frac{n^2}{n^2 + 1} : n \in \mathbb{N} \right\}.$$

Find $\sup A$ and $\inf A$ with proving them.

4. **(10 points)** Use Definition to prove that

$$\lim_{n \rightarrow \infty} \frac{n}{\sqrt{n^2 + 1}} = 1.$$

5. (10 points) Assume that $x_n \rightarrow -2$ as $n \rightarrow \infty$. Show that

$$\frac{1}{1 + (x_n)^2} \rightarrow \frac{1}{5} \quad \text{as } n \rightarrow \infty.$$

6. (10 points) Let $\{x_n\}$ be a sequence in \mathbb{R} . Prove that

$\{x_n\}$ is Cauchy if and only if $\{x_n\}$ converges (some point in \mathbb{R})

7. (10 points) Let A be a nonempty subset of \mathbb{R} . Assume that A is open. Prove that

$$\inf A \notin A \quad \text{and} \quad \sup A \notin A.$$

8. (10 points) Use definition to prove that

$$\lim_{x \rightarrow -1} \frac{x}{x+2} = -1.$$

9. (10 points) Use definition to prove that

$$\lim_{x \rightarrow 2^-} \frac{1}{\sqrt{2-x}} = +\infty.$$

10. (10 points) Let f and g be real functions from a set E to \mathbb{R} . Assume that

$$f(x) \leq g(x) \quad \text{for all } x \in E.$$

Let a is a limit point of E . Prove that if $f(x) \rightarrow \infty$ as $x \rightarrow a$, then

$$g(x) \rightarrow \infty \text{ as } x \rightarrow a.$$

Extra (5 points) Let X' represent the set of all limit points of X . Let A and B be two subset of \mathbb{R} . Show that

$$(A \cup B)' = A' \cup B'.$$