Suan Sunandha Rajabhat University
Faculty of Education, Branch of Mathematics
Midterm Examination, Semester 2/2016

ID Subject MAT1202	Course Name Set Theory	Test Time 5pm - 8pm Mon 7 Mar 2017	Full Scores 105 points 30%

Name.. **ID**.................................. **Section**..............

Direction

1. 10 questions and 10 pages.

2. Write obviously your name, id and section all pages.

3. Without calculators and communication tools.

4. Don't take text books and others come to the test room.

5. Cannot answer sheets out of test room.

6. Deliver to the staff if you make a mistake in the test room.

Signature

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Lecturer: Thanatyod Jampawai, Ph.D.

	1	2	3	4	5	6	7	8	9	10	

1. **(10 points)** Write answers in the right blanks

   1.1 Let $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8\}$, $A - B = \{1, 2, 3\}$ and $B - A = \{4, 5\}$.
        Find $A \cap B$ _____

   1.2 Let $A = \{\varnothing, \{\varnothing\}\}$. Find $\mathcal{P}(A) - A$. _____

   1.3 Find $(A \times B) - (B \times A)$. If $A = \{1, 2, 3\}$ and $B = \{2, 3, 4\}$. _____

   1.4 Let $r = \{(x, y) \in \mathbb{R} \times \mathbb{R} : |x| + |y| = 1\}$. Compute $Dom(r) \cap Ran(r)$. _____

   1.5 Let r and s be relations on $A = \{1, 2, 3\}$. Suppose $r = \{(1, 2), (2, 2), (3, 3)\}$
        and $s = \{(a, b)\}$. If $r \cup s$ is reflexive, what is (a, b). _____

   1.6 Define $f(x) = \dfrac{x + 1}{x - 1}$. If $a \in f^{-1}(\{2\})$, what is a. _____

   1.7 Let $f\left(\dfrac{x - 1}{x + 1}\right) = x$. Estimate $f(f^{-1}(-2) + f(2))$ _____

   1.8 Define $f(x + 1) = x - 1$ and $g = \{(1, 2), (2, 3), (3, 4)\}$. Find $g(f^{-1}(0))$ _____

   1.9 Define $f(x) = \begin{cases} 3x & \text{if } x^2 < 1 \\ -3x & \text{if } x^2 \geq 1 \end{cases}$. Compute $f^{-1}(\{-1, 0, 1\})$ _____

   1.10 Let $A = \{a, b\}$. Find a choice functions of A. _____

2. Explain your answers

    2.1 **(8 points)**

       (a) Write out set A in builder form (only notation).

$$A = \{0.1, 0.04, 0.009, 0.0016, ...\}$$

       (b) Let $S = \{1, 2, 3, 4, 5\}$. List elements of set

$$B = \{(x, y) \in S \times S : x \mid (x + y)\}$$

    2.2 **(7 points)** Let A and B be sets. Prove that

$$A - B = B - A \quad \text{if and only if} \quad A = B$$

3. Explain your answers

   3.1 **(5 points)** Draw Haase diagram of $\{1\}, \{2\}, \{3\}, \{5\}, \{1,2\}, \{2,4\}, \{1,2,3\}, \{1,2,4,5\}$

   3.2 **(5 points)** Let $B = \{1,2,3,4\}$ and $A \in \mathcal{P}(B)$. If $A \cap \{3\} = \varnothing$, list all possible sets A.

   3.3 **(5 points)** Let A and B be sets. Show that

   $$\text{if} \quad \mathcal{P}(A) = \mathcal{P}(B), \quad \text{then} \quad A = B.$$

4. Explain your answers

    4.1 **(5 points)** Let A and B be sets. Prove that

$$(B \times A) \cap (C \times A) = (B \cap C) \times A$$

    4.2 **(6 points)** Let

$$r = \{(x, y) \in \mathbb{R} \times \mathbb{R} : y \geq x^2\} \quad \text{and} \quad s = \{(x, y) \in \mathbb{R} \times \mathbb{R} : x^2 + y^2 = 1\}.$$

    Find $Dom(r \cap s)$ and $Ran(r \cap s)$.

5. Let r be a relation on a set A.

    5.1 **(5 points)** Write definitions of five properties of r.

       1. Reflexive:

       2. Symmetric:

       3. Transitive:

       4. Antisymmetric:

       5. Total:

    5.2 **(6 points)** Show that

$$\text{if } r \text{ is symmetric on } A, \text{ then } \quad r \cup r^{-1} \text{ is symmetric on } A.$$

6. Let $A = \{1, 2, 3, ..., 20\}$ and $(A, |)$ be a poset. Let $B = \{2, 4, 5\}$

    6.1 **(4 points)** Draw diagram of the poset $(A, |)$

    6.2 **(2 points)** Compute minimal and maximal elements of B.

    6.3 **(3 points)** Find B_L and B_U

    6.4 **(2 points)** Compute $\inf B$ and $\sup B$

7. **(8 points)** Let $f(x) = x|x|$. Show that f is injective and find $f^{-1}(x)$.

8. **(8 points)** Let $f : A \rightarrow B$ be a function and U and V be subsets of B. **Prove** or **disprove** the statement

$$f^{-1}(U - V) = f^{-1}(U) - f^{-1}(V)$$

9. Let $g(x) = \begin{cases} 1 - x & \text{if } x \leq 1 \\ x - 1 & \text{if } x > 1 \end{cases}$ and $f(x) = \begin{cases} g(x) & \text{if } x > 0 \\ g(-x) & \text{if } x \leq 0 \end{cases}$ Find

9.1 **(1 points)** $g \circ f(-1)$

9.2 **(2 points)** $f_*(\{-1, -\frac{1}{2}, 0, \frac{1}{2}, 1\})$

9.3 **(2 points)** $f^*(\{1, 2\})$

9.4 **(3 points)** $(f \circ g)(x)$

10. Explain your answers

    10.1 **(4 points)** Let $\quad f(x) = \dfrac{1}{1 + \frac{1}{x}} \quad$ and $\quad g(x) = \dfrac{x}{1 + x}$. Are f and g equal ? Verify your answer.

    10.2 **(4 points)** If a student does not understand about word **SET**, how do you explain.