Suan Sunandha Rajabhat University
Faculty of Education, Branch of Mathematics
Midterm Examination, Semester 1/2016

ID Subject	Course Name	Test Time	Full Scores
MAT2303	Abstract Albegra	5pm - 8pm Wed 5 Oct 2016	105 points 30%

Name... **ID**.................................... **Section**...............

Direction

1. 12 questions and 11 pages.
2. Write obviously your name, id and section all pages.
3. Without calculators and communication tools.
4. Don't take text books and others come to thetest room.
5. Cannot answer sheets out of test room.
6. Deliver to the staff if you make a mistake in the test room.

Signature

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Lecturer: Thanatyod Jampawai, Ph.D.

No.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**Total**
Score													

1. **(10 points)** Write answers in the right blanks

   1.1 What is the **inverse** of $\bar{3}$ in $\mathbb{Z}_7$? ________________

   1.2 What is the **inverse** of $(135)(42)$ in S_5 ? ________________

   1.3 Compute the **order** of $(132)(31)(24)$ in S_4 ________________

   1.4 Compute the **order** of $\overline{17}$ in $\mathbb{Z}_{20}^{\times}$ ________________

   1.5 Find a **generator** of $\mathbb{Z}_5^{\times}$ ________________

   1.6 Write out **elements** of $\langle\bar{4}\rangle$ in $\mathbb{Z}_{16}$ ________________

   1.7 Find the **number of all generators** for $\mathbb{Z}_{1000}$ ________________

   1.8 Write out elements of the **left coset** $(12)\,\langle(31)\rangle$ in S_3. ________________

   1.9 Compute the **index** $[\mathbb{Z}_{25}^{\times} : \langle\bar{7}\rangle]$ ________________

   1.10 What is the **inverse** of $\langle\bar{5}\rangle + \bar{2}$ in a quotient group $\mathbb{Z}_{13}/\langle\bar{5}\rangle$? ________________

2. **(6 points)** Define $a * b = a + b - 7$ for all $a, b \in \mathbb{Z}$. Prove that $(\mathbb{Z}, *)$ is a group.

3. Let G be a group. Prove that

   3.1 **(4 points)** if $x^2 = e$ for all $x \in G$, then G is abelian.

   3.2 **(3 points)** $(ab)^{-1} = b^{-1}a^{-1}$ for all $a, b \in G$.

4. In symmetric groups

4.1 **(4 points)** Write the **cycle decomposition** of each element

(a) of **order 4** in S_4

(b) of **order 2** in S_6

4.2 **(4 points)** Compute the orders of

(a) $(1\ 2\ 3)(4\ 3)(4\ 5)(6\ 7)$ in S_8

(b) $(1\ 7\ 9)(2\ 10\ 3)(7\ 8)$ in S_{10}

5. **(10 points)** Let $H = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : ad - bc = 1 \right\}$. Prove that H is a subgroup of $GL_2(\mathbb{R})$.

6. **(5 points)** Let G be a group. **Prove or disprove** that

$$\text{if} \quad H \leq G \quad \text{and} \quad K \leq G, \text{ then} \quad H \cup K \leq G.$$

7. In $\mathbb{Z}_{18}^{\times}$ with multiplication

   7.1 **(5 points)** Find **all generators**

   7.2 **(5 points)** Find **all subgroups** by Lagrance's theorem

8. **(8 points)** Write the **Lattice diagram** of $\mathbb{Z}_{36}$ (Write each subgroup by $\langle a \rangle$)

9. Explain your answers

   9.1 **(8 points)** Find **all normal subgroups** of S_3

   9.2 **(5 points)** Let M and N be subgroups of a group G. Prove that

$$M \trianglelefteq G \text{ and } N \trianglelefteq G \longrightarrow M \cap N \trianglelefteq G$$

10. Explain your answers

10.1 **(3 points)** Write out **elements** of $\mathbb{Z}/5\mathbb{Z}$

10.2 **(5 points)** Write inverses of each element in quoteint group $\mathbb{Z}_{25}^{\times}/\langle\bar{7}\rangle$

11. Let $G_1 = \mathbb{Z} \times \mathbb{Z}$ be a group with addition and $G_2 = \mathbb{Q}^+$ a group with multiplication. Define

$$\varphi : G_1 \to G_2 \quad \text{by} \quad \varphi(a, b) = 2^{a+b} \quad \text{for all } a, b \in \mathbb{Z}$$

11.1 **(4 points)** Prove that φ is homormorphism

11.2 **(4 points)** Is φ isomorphism ? Verfy your answer.

11.3 **(4 points)** Find $Ker(\varphi)$ and $Im(\varphi)$

12. Explain your answers

 12.1 **(4 points)** Show that $\mathbb{Z}_2 \times \mathbb{Z}_3$ is isomorphic to $\mathbb{Z}_6$

 12.2 **(4 points)** In $\mathbb{Z}_4$, find T_0, T_1, T_2 and T_3 and a subgroup H of S_4 such that $\mathbb{Z}_4 \cong H$ by Cayley's theorem.