Suan Sunandha Rajabhat University
Faculty of Education, Branch of Mathematics
Midterm Examination, Semester 2/2016

ID Subject	**Course Name**	**Test Time**	**Full Scores**
MAT12305	Linear Algebra	5pm - 8pm Wed 8 Mar 2017	106 points 30%

Name.. **ID**.................................... **Section**...............

Direction

1. 10 questions and 10 pages.
2. Write obviously your name, id and section all pages.
3. Can use a calculator(s) but can not use communication tools.
4. Don't take text books and others come to the test room.
5. Cannot answer sheets out of test room.
6. Deliver to the staff if you make a mistake in the test room.

Signature

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Lecturer: Thanatyod Jampawai, Ph.D.

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	

1. **(10 points)** Write answers in the right blanks

1.1 Find a **linear equation** in the variables x, y and z that has the general solution $x = s + t$, $y = 2t$ and $z = s - 1$. ______________

1.2 Let $\begin{bmatrix} 1 & a \\ b & 2 \end{bmatrix} \xrightarrow{R_2 - 2R_1} \begin{bmatrix} 1 & 3 \\ 1 & c \end{bmatrix}$. Compute $a + b + c$. ______________

1.3 If $A \begin{bmatrix} 2 & 4 \\ 6 & 10 \end{bmatrix} = 2I$, what is A^{-1}. ______________

1.4 Let $A = \begin{bmatrix} -1 & 2 & 3 \\ 2 & -3 & 1 \\ 0 & 1 & 2 \end{bmatrix}$ and LU-decomposition be $A = LU$. Find L. ______________

1.5 Compute $\begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 4 \end{vmatrix}$. ______________

1.6 Let A and B be an 2×2 matrices. If $\det(A) = -2$ and $\det(B) \neq 0$, find $\det(3B^T A^2 B^{-1})$ ______________

1.7 If $\text{adj}(A) = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$, what is A^{-1}. ______________

1.8 Let $B = \{(1, 1), (-1, 0)\}$ be a basis for $\mathbb{R}^2$ and $v = (2, 3)$. Find $(v)_B$. ______________

1.9 Let $A = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$. Compute A^{2560}. ______________

1.10 Let $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 2 & 4 \end{bmatrix}$. Find nullity$(A)$. ______________

2. Explain your answers

   2.1 **(6 points)** Using Gussian elimination to solve linear system

$$\begin{cases} x_1 + x_2 + 2x_3 = 1 \\ 2x_1 + 3x_2 - x_3 = 2 \\ -x_1 - 3x_2 + 8x_3 = -1 \end{cases}$$

   2.2 **(6 points)** Let A and B be an 2×2 matrices satisfying

$$2A + B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad \text{and} \quad A - B = \begin{bmatrix} 2 & 4 \\ -3 & 2 \end{bmatrix}$$

   Compute $(AB)^{-1}$

3. **(10 points)** Using Gussian elimination to solve the non-linear system by substitution and verify your answers.

$$\begin{cases} \ln x & - & \ln y & + & \ln z & = & 3 \\ 3\ln x & - & 4\ln y & + & \ln z & = & 1 \\ \ln x & + & 2\ln y & - & \ln z & = & 3 \end{cases}$$

4. **(6 points)** Let

$$A = \begin{bmatrix} 8 & x+y & x-y+2z \\ -1 & 9 & -5 \\ 3 & y-2z & 8 \end{bmatrix}$$

be a symmetric matrix. What are x, y and z ?

5. Let $A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 2 & 1 & 0 & 1 \\ 0 & 3 & -5 & 1 \\ 1 & 4 & 2 & -13 \end{bmatrix}$.

   5.1 **(7 points)** Compute A^{-1}.

   5.2 **(3 points)** Use (4.1) to solve linear system in form $A\boldsymbol{x} = \boldsymbol{b}$

$$\begin{cases} x_1 & & + & x_3 & & & = & 1 \\ 2x_1 & + & x_2 & & & + & x_4 & = & 1 \\ & & 3x_2 & - & 5x_3 & + & x_4 & = & 1 \\ x_1 & + & 4x_2 & + & 2x_3 & - & 13x_4 & = & 1 \end{cases}$$

6. **(10 points)** Solve the folowing linear system by LU-decomposition.

$$\begin{cases} 2x_1 & + & x_2 & + & 3x_3 & = & -6 \\ 4x_1 & + & 3x_2 & + & 5x_3 & = & -8 \\ -6x_1 & + & 7x_2 & + & x_3 & = & 18 \end{cases}$$

7. Explain your answers

7.1 **(6 points)** Compute determinant of $A = \begin{bmatrix} 1 & 0 & 1 & 2 \\ -1 & 1 & 0 & 3 \\ 2 & 0 & 1 & 1 \\ 0 & 1 & 2 & 3 \end{bmatrix}$ by EROs or Cofactor expansion.

7.2 **(6 points)** Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ and $B = \begin{bmatrix} 2a_{11} & a_{21} \\ 2a_{12} & a_{22} \end{bmatrix}$. If $\det(B) = -4$, find

(a) $\det(A)$

(b) $\det(3A^2B)$

(c) $\det(-B^T(2A^2)^{-1})$

8. Explain your answers

8.1 **(6 points)** Let $A = \begin{bmatrix} 1 & -1 & 1 & 3 \\ 0 & 1 & -3 & 3 \\ 1 & 0 & 0 & 3 \\ 1 & -1 & 2 & 3 \end{bmatrix}$. If $A^{-1} = [a^*_{ij}]$, find $a^*_{31} + a^*_{42}$.

8.2 **(6 points)** Let $A\boldsymbol{x} = \boldsymbol{b}$ be a linear system where $A = [a_{ij}]_{3\times 3}$, $\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ and $\boldsymbol{b} = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$. If we can compute x_1 and x_2 by Cramer's rule,

$$x_1 = \frac{\begin{vmatrix} 2 & -1 & 1 \\ 3 & 0 & 2 \\ 1 & 1 & 0 \end{vmatrix}}{|A|} \qquad \text{and} \qquad x_2 = \frac{\begin{vmatrix} 1 & 2 & 1 \\ 1 & 3 & 2 \\ 0 & 1 & 0 \end{vmatrix}}{|A|},$$

find x_3.

9. Explain your answers

   9.1 **(6 points)** Let $W = \{(x, y, z) : xyz = 0\}$. Check that W is a subspace of $\mathbb{R}^3$ or NOT.

   9.2 **(6 points)** Show that $B = \{(1, -1, 1), (1, 1, 0), (0, 1, 1)\}$ is a basis for $\mathbb{R}^3$. If $v = (1, 2, 3)$, find $(v)_B$

10. Explain your answers

 10.1 **(6 points)** Find bases for the row space and column space of

$$A = \begin{bmatrix} 1 & 2 & 0 & 2 \\ -2 & -5 & 5 & 6 \\ 0 & -3 & 15 & 18 \\ 0 & -2 & 10 & 8 \\ 3 & 6 & 0 & 6 \end{bmatrix}$$

 consisting entirely of row vector from A.

 10.2 **(6 points)** Let

$$A = \begin{bmatrix} 1 & 2 & -3 & 0 & 1 & 0 \\ 1 & 3 & 0 & 0 & -1 & 3 \\ -2 & -4 & 6 & 0 & -2 & 0 \\ 0 & 0 & 2 & 2 & 0 & 2 \end{bmatrix}$$

 Find Rank(A) and a basis of Null(A).