Suan Sunandha Rajabhat University
Faculty of Education, Division of Mathematics
Midterm Examination, Semester 2/2017

ID Subject	Course Name	Test Time	Full Scores
MAT12305	Linear Algebra	1pm - 4pm Fri 2 Mar 2018	105 points 30%

Name.. **ID**.................................. **Section**...............

Direction

1. 10 questions and 11 pages.
2. Write obviously your name, id and section all pages.
3. Can use a calculator(s) but can not use communication tools.
4. Don't take text books and others come to the test room.
5. Cannot answer sheets out of test room.
6. Deliver to the staff if you make a mistake in the test room.

Signature

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Lecturer: Thanatyod Jampawai, Ph.D.

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	

1. **(10 points)** Write answers in the right blanks

1.1 Let $\{(2t-1, 3s+1, 1-t+s) : t, s \in \mathbb{R}\}$ is the solution set of $ax+by+cz=1$ where $a, b, c \in \mathbb{Z}$. Find $a+b+c$ ________________

1.2 Let $\left[\begin{array}{cc|c} 1 & 2 & 1 \\ 0 & 1 & 3 \end{array}\right]$ be a augmented matrix with variables x, y. Find $x+y$ ________________

1.3 Let A be a mtrix obtained from B by R_{12} and $R_2 - 2R_1$, respectively.
If $B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, find A ________________

1.4 Let $A = \begin{bmatrix} 1 & 3 \\ a & k \end{bmatrix}$. If A is symmetric and is not invertible. What is k ? ________________

1.5 Let A be a 3×3 matrix such that $A + A^T = 0$.
Compute **sum of all entries** in A. ________________

1.6 Use Cramer's rule to sove a linear system with two variable, x_1, x_2,

when $x_1 = \dfrac{\begin{vmatrix} 1 & a \\ 5 & b \end{vmatrix}}{\begin{vmatrix} 1 & 2 \\ 1 & 4 \end{vmatrix}}$. Find x_2 ________________

1.7 If $A^{-2} = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} \sin\theta & \cos\theta \\ \cos\theta & -\sin\theta \end{bmatrix}^{101}$ where $\theta \in \mathbb{R}$.

Compute $\det(7(A^2B^{-5})^{2018})$. ________________

1.8 Let $\begin{vmatrix} a+2c & b+2d \\ 3c & 3d \end{vmatrix} = 15$. Compute $\begin{vmatrix} c & d \\ a & b \end{vmatrix}$. ________________

1.9 Let $B = \{(1,1), (1,0)\}$ be a basis for $\mathbb{R}^2$ and $v = (3,4)$. Find $(v)_B$. ________________

1.10 Let A be 5×8 matrix. If $\text{rank}(A) = 2$, find $\text{nullity}(A)$ ________________

2. Explain your answers

2.1 **(6 points)** Using **Guss-Jardan elimination** to solve linear system

$$\begin{cases} x_1 + x_2 + 2x_3 + 2x_4 = 1 \\ -x_1 - 2x_2 + x_3 + x_4 = 2 \\ -x_2 + 3x_3 + 3x_4 = 3 \\ -3x_1 - 2x_2 - 3x_3 - 3x_4 = 0 \end{cases}$$

2.2 **(6 points)** If the following linear system is **consistent** ?

$$\begin{cases} x + 2y + 3z = b \\ 2x + y + 3z = 3 - b \\ 7x + 11y + 18z = 15 \end{cases}$$

What is b ?

3. Explain your answers

3.1 **(6 points)** Let $A = \begin{bmatrix} 3 & 4 & 3 \\ 2 & 0 & y \\ 1 & 1 & 2 \end{bmatrix}$, $\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ and $\boldsymbol{b} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$. If

$$\left[\begin{array}{ccc|ccc} 3 & 4 & 3 & 1 & 0 & 0 \\ 2 & 0 & y & 0 & 1 & 0 \\ 1 & 1 & 2 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 9 & z & -36 \\ 0 & 1 & 0 & -5 & -3 & 21 \\ 0 & 0 & 1 & -2 & -1 & 8 \end{array}\right],$$

(a) **(3 points)** find y and z

(b) **(3 points)** solve linear system $A\boldsymbol{x} = \boldsymbol{b}$

3.2 **(6 points)** Let $A = \begin{bmatrix} 1 & a \\ 1 & b \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$. If $(A+B)^2 = A^2 + 2AB + B^2$. Find $(A^{-1} + A^T)^2$

4. **(10 points)** Solve the folowing linear system by LU-decomposition.

$$\begin{cases} x_1 - 3x_2 + 5x_3 = 2 \\ 2x_1 - 5x_2 + x_3 = -3 \\ -x_1 + x_2 + 2x_3 = 1 \end{cases}$$

5. Explain your answers

   5.1 **(5 points)** Use the **determinant definition** to evaluate $\begin{vmatrix} 0 & 0 & 0 & 3 & 4 \\ 0 & 5 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 6 & 0 \\ 7 & 0 & 0 & 0 & 0 \end{vmatrix}$

   5.2 **(5 points)** Let A be a 2×2 matrix such that $\det(A) = 3$. If $A - 3I$ is not invertible, find $\det(A + 3I)$ where I is identity 2×2 matrix.

6. Explain your answers

6.1 **(6 points)** Let $A = \begin{bmatrix} a+3d & b+3e & c+3f \\ 2d-a & 2e-b & 2f-c \\ 2x & 2y & 2z \end{bmatrix}$ and $B = \begin{bmatrix} a & d & x \\ b & e & y \\ c & f & z \end{bmatrix}$.
If $\det(A) = 100$, find $\det[3A^4(5B)^{-1}]$

6.2 **(6 points)** Let $\text{adj}(A) = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 3 \\ 3 & 5 & 5 \end{bmatrix}$ and $\det(A) > 0$.

(a) **(2 points)** Find $\det(A)$

(b) **(4 points)** Find A

7. Explain your answers

7.1 **(5 points)** Let $W = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : a+b+c=d \right\}$. Check that W is a **subspace** of $M_{22}(\mathbb{R})$ or NOT.

7.2 **(4 points)** Let A be an $n \times n$ matrix. Show that

$$W = \{\boldsymbol{x} \in \mathbb{R}^n : A\boldsymbol{x} = \mathbf{0}\}$$

is a subspace of $\mathbb{R}^n$.

8. Explain your answers

8.1 **(6 points)** Let $\mathcal{B} = \{(1,1,1),(1,0,1),(1,1,0)\}$ and $v = (2,1,8)$.

(a) **(3 points)** Show that $\mathcal{B}$ is a basis for $\mathbb{R}^3$

(b) **(3 points)** Find $(v)_{\mathcal{B}}$

8.2 **(5 points)** Let $S = \{xe^x, \sin x, \cos x\}$. Determine whether S is linearly independent.

9. Explain your answers

   9.1 **(5 points)** Determine whether $\boldsymbol{p} = 3x^2 + 2x + 1$ is in $\text{span}\{1 + x, 1 + x^2, 1 - x^2\}$

   9.2 **(5 points)** Let $A = \begin{bmatrix} x & 1 & 0 \\ 0 & -x & 3 \\ 0 & 0 & -x \end{bmatrix}$ where $x > 0$. If $(I - A^{-1})$ is a singular matrix. Find $\det(A + I)$

10. Explain your answers

10.1 **(5 points)** Find **bases** for the **row space** and **column space**, rank and nullity of

$$A = \begin{bmatrix} 1 & 2 & 0 & 1 & 3 & 4 \\ 2 & 3 & 2 & 3 & -1 & 2 \\ 3 & 5 & 2 & 4 & 2 & 6 \\ 0 & -1 & 2 & 1 & -7 & -6 \\ 5 & 9 & 2 & 6 & 8 & 13 \end{bmatrix}$$

10.2 **(5 points)** In mathayom class room, there is a question from a student said that for any square matrices A and B,

$$\text{if } AB = 0, \text{ then } A = 0 \text{ or } B = 0$$

TRUE or **FALSE**. If you are a teacher in this class, how you will answer for the question.