QUIZ 1 : MAT2303 ABSTRACT ALGEBRA

TOPIC	Groups & Subgroups	**SCORE**	10 points
QUIZ TIME	Wed 7 Sep 2016, 4th Week, Semester 1/2016		
TEACHER	Thanatyod Jampawai, Ph.D., Faculty of Education, Suan Sunandha Rajabhat University		

NAME.. **ID**.................................... **SECTION**.......................

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1. Define $a * b = 13ab$ for all $a, b \in \mathbb{Q}^+$. Prove that $(\mathbb{Q}^+, *)$ is a group. (3 points)

2. Compute **inverses** and **orders** for each element in the following groups. (4 points)

   2.1 $\bar{4}$ in $(\mathbb{Z}_6, +)$

   2.2 $\bar{7}$ in $(\mathbb{Z}_{15}^{\times}, \cdot)$

   2.3 $\begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$ in $(GL_2(\mathbb{R}), \cdot)$

   2.4 $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 1 & 4 \end{pmatrix}$ in S_4

3. Find all subgroups of $(\mathbb{Z}_7^{\times}, \cdot)$ and verify your answers. (3 points)

ANSWERS QUIZ 1 : MAT2303 ABSTRACT ALGEBRA

TOPIC	Groups & Subgroups	**SCORE**	10 points
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1. Define $a * b = 13ab$ for all $a, b \in \mathbb{Q}^+$. Prove that $(\mathbb{Q}^+, *)$ is a group. (3 points)

Proof. First, let $a, b, c \in \mathbb{Q}^+$. Then

$$(a * b) * c = (13ab) * c = 13(13ab)c = 13(a)(13bc) = a * (13bc) = a * (b * c).$$

So, $*$ is associative on $\mathbb{Q}^+$. Next, let $a \in \mathbb{Q}^+$. We obtain

$$a * \frac{1}{13} = 13a(\frac{1}{13}) = a = 13(\frac{1}{13})a = \frac{1}{13} * a.$$

Thus, $\frac{1}{13}$ is an identity. Finally, we will prove that all elements in $\mathbb{Q}^+$ have inverses. Let $a \in \mathbb{Q}^+$. Since a in a nonzero rational number, $\frac{1}{169a}$ is a positive rational number. We get

$$a * (\frac{1}{169a}) = 13a(\frac{1}{169a}) = \frac{1}{13} = 13(\frac{1}{169a})a = \frac{1}{169a} * a.$$

Hence, $\frac{1}{169a}$ ia an inverses of a. Therefore, $(\mathbb{Q}^+, *)$ is a group. $\square$

2. Compute **inverses** and **orders** for each element in the following groups. (4 points)

Elements	**Inverses**	Reasons	**Orders**	Reason
$\bar{4}$	$\bar{2}$	$\bar{4} + \bar{2} = \bar{0}$	3	$\bar{4} + \bar{4} + \bar{4} = \bar{0}$
$\bar{7}$	$\bar{13}$	$\bar{7} \cdot \bar{13} = \bar{1}$	4	$\bar{7} \cdot \bar{7} \cdot \bar{7} \cdot \bar{7} = \bar{1}$
$\begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	2	$\begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 1 & 4 \end{pmatrix}$	$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 2 & 4 \end{pmatrix}$	$\begin{pmatrix} 2 & 3 & 1 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix}$	3	$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 1 & 4 \end{pmatrix}^3 = (1)$

3. Find all subgroups of $(\mathbb{Z}_7^\times, \cdot)$ and verify your answers. (3 points)

Subsets	**Subgroups**	Reasons
$\{\bar{1}\}$	**Yes**	$\bar{1} \cdot \bar{1} = \bar{1}$
$\{\bar{1}, \bar{2}\}$	No	$\bar{2} \cdot \bar{2} = \bar{4}$
$\{\bar{1}, \bar{3}\}$	No	$\bar{3} \cdot \bar{3} = \bar{2}$
$\{\bar{1}, \bar{4}\}$	No	$\bar{4} \cdot \bar{4} = \bar{2}$
$\{\bar{1}, \bar{5}\}$	No	$\bar{5} \cdot \bar{5} = \bar{4}$
$\{\bar{1}, \bar{6}\}$	**Yes**	$\bar{6} \cdot \bar{6} = \bar{1}$
$\{\bar{1}, \bar{2}, \bar{3}\}$	No	$\bar{2} \cdot \bar{2} = \bar{4}$
$\{\bar{1}, \bar{2}, \bar{4}\}$	**Yes**	$\bar{2} \cdot \bar{2} = \bar{4}$ $\bar{2} \cdot \bar{4} = \bar{1}$ $\bar{4} \cdot \bar{4} = \bar{2}$
$\{\bar{1}, \bar{2}, \bar{5}\}$	No	$\bar{2} \cdot \bar{2} = \bar{4}$
$\{\bar{1}, \bar{2}, \bar{6}\}$	No	$\bar{2} \cdot \bar{2} = \bar{4}$
$\{\bar{1}, \bar{3}, \bar{4}\}$	No	$\bar{3} \cdot \bar{3} = \bar{2}$
$\{\bar{1}, \bar{3}, \bar{5}\}$	No	$\bar{3} \cdot \bar{3} = \bar{2}$
$\{\bar{1}, \bar{3}, \bar{6}\}$	No	$\bar{3} \cdot \bar{3} = \bar{2}$
$\{\bar{1}, \bar{4}, \bar{5}\}$	No	$\bar{5} \cdot \bar{5} = \bar{4}$
$\{\bar{1}, \bar{4}, \bar{6}\}$	No	$\bar{4} \cdot \bar{6} = \bar{3}$
$\{\bar{1}, \bar{5}, \bar{6}\}$	No	$\bar{5} \cdot \bar{5} = \bar{4}$

Subsets	**Subgroups**	Reasons
$\{\bar{1}, \bar{2}, \bar{3}, \bar{4}\}$	No	$\bar{3} \cdot \bar{4} = \bar{5}$
$\{\bar{1}, \bar{2}, \bar{3}, \bar{5}\}$	No	$\bar{5} \cdot \bar{5} = \bar{4}$
$\{\bar{1}, \bar{2}, \bar{3}, \bar{6}\}$	No	$\bar{3} \cdot \bar{6} = \bar{4}$
$\{\bar{1}, \bar{3}, \bar{4}, \bar{5}\}$	No	$\bar{4} \cdot \bar{5} = \bar{6}$
$\{\bar{1}, \bar{3}, \bar{4}, \bar{6}\}$	No	$\bar{3} \cdot \bar{4} = \bar{5}$
$\{\bar{1}, \bar{4}, \bar{5}, \bar{6}\}$	No	$\bar{5} \cdot \bar{6} = \bar{2}$
$\{\bar{1}, \bar{3}, \bar{5}, \bar{6}\}$	No	$\bar{5} \cdot \bar{6} = \bar{2}$
$\{\bar{1}, \bar{3}, \bar{4}, \bar{5}\}$	No	$\bar{5} \cdot \bar{4} = \bar{6}$
$\{\bar{1}, \bar{2}, \bar{5}, \bar{6}\}$	No	$\bar{5} \cdot \bar{5} = \bar{4}$
$\{\bar{1}, \bar{2}, \bar{4}, \bar{6}\}$	No	$\bar{4} \cdot \bar{6} = \bar{3}$
$\{\bar{1}, \bar{2}, \bar{4}, \bar{5}\}$	No	$\bar{4} \cdot \bar{5} = \bar{6}$
$\{\bar{1}, \bar{2}, \bar{3}, \bar{4}, \bar{5}\}$	No	$\bar{4} \cdot \bar{5} = \bar{6}$
$\{\bar{1}, \bar{2}, \bar{3}, \bar{4}, \bar{6}\}$	No	$\bar{3} \cdot \bar{4} = \bar{5}$
$\{\bar{1}, \bar{2}, \bar{3}, \bar{5}, \bar{6}\}$	No	$\bar{5} \cdot \bar{5} = \bar{4}$
$\{\bar{1}, \bar{2}, \bar{4}, \bar{5}, \bar{6}\}$	No	$\bar{2} \cdot \bar{5} = \bar{3}$
$\{\bar{1}, \bar{3}, \bar{4}, \bar{5}, \bar{6}\}$	No	$\bar{5} \cdot \bar{6} = \bar{2}$
$\mathbb{Z}_7^\times$	**Yes**	

Thus, All subgroups of $(\mathbb{Z}_7^\times, \cdot)$ are $\{\bar{1}\}$, $\{\bar{1}, \bar{6}\}$, $\{\bar{1}, \bar{2}, \bar{4}\}$ and $\mathbb{Z}_7^\times$.

QUIZ 2 : MAT2303 ABSTRACT ALGEBRA

TOPIC Subgroups & Cyclic groups **SCORE** 10 points
QUIZ TIME Wed 14 Sep 2016, 6th Week, Semester 1/2016
TEACHER Thanatyod Jampawai, Ph.D., Faculty of Education, Suan Sunandha Rajabhat University

NAME.. **ID**.................................... **SECTION**.......................

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1. Let $H = \left\{ \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} : ab \neq 0 \right\}$. Prove that H is a subgroup of $GL_2(\mathbb{R})$. (3 points)

2. Find all generators of the following groups. (3 points)

   2.1 $(\mathbb{Z}_{36}, +)$

   2.2 $(\mathbb{Z}_{25}^{\times}, \cdot)$

3. Find all subgroups of the following groups by Lagrange's theorem. (4 points)

   3.1 $(\mathbb{Z}_{18}, +)$

   3.2 $(\mathbb{Z}_{25}^{\times}, \cdot)$

QUIZ 2 : MAT2303 ABSTRACT ALGEBRA

TOPIC Subgroups & Cyclic groups **SCORE** 10 points
QUIZ TIME Wed 14 Sep 2016, 6th Week, Semester 1/2016
TEACHER Thanatyod Jampawai, Ph.D., Faculty of Education,
Suan Sunandha Rajabhat University
NAME.. **ID**.................................... **SECTION**.......................

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1. Let $H = \left\{ \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} : ab > 0 \right\}$. Prove that H is a subgroup of $GL_2(\mathbb{R})$. (3 points)

2. Find all generators of the following groups. (3 points)

   2.1 $(\mathbb{Z}_{48}, +)$ 2.2 $(\mathbb{Z}_{25}^{\times}, \cdot)$

3. Find all subgroups of the following groups by Lagrange's theorem. (4 points)

   3.1 $(\mathbb{Z}_{24}, +)$ 3.2 $(\mathbb{Z}_{25}^{\times}, \cdot)$

ANSWER QUIZ 2 : MAT2303 ABSTRACT (SEC1)

TOPIC	Subgroups & Cyclic groups	**SCORE**	10 points
QUIZ TIME	Wed 14 Sep 2016, 6th Week, Semester 1/2016		
TEACHER	Thanatyod Jampawai, Ph.D., Faculty of Education, Suan Sunandha Rajabhat University		

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1. Let $H = \left\{ \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} : ab \neq 0 \right\}$. Prove that H is a subgroup of $GL_2(\mathbb{R})$.

Proof. We first choose $a = b = 1$, $ab = 1 \neq 0$. So, $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ belongs to H.

Next, we will show that H is closed. Let $A = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$ and $B = \begin{bmatrix} x & 0 \\ 0 & y \end{bmatrix}$ be elements in H. Then

$$AB = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \begin{bmatrix} x & 0 \\ 0 & y \end{bmatrix} = \begin{bmatrix} ax & 0 \\ 0 & by \end{bmatrix}$$

Since $A, B \in H$, $ab \neq 0$ and $xy \neq 0$. We conclude that $(ax)(by) = (ab)(xy) \neq 0$. Thus, $AB \in H$.
Finally, let $A = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$ be in H. Then $ab \neq 0$. It follows that $a \neq 0$ and $b \neq 0$. Choose $A^{-1} = \begin{bmatrix} \frac{1}{a} & 0 \\ 0 & \frac{1}{b} \end{bmatrix}$.
Since $a \neq 0$ and $b \neq 0$, $\frac{1}{ab}$ is non zeoro. Then

$$AA^{-1} \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \begin{bmatrix} \frac{1}{a} & 0 \\ 0 & \frac{1}{b} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{a} & 0 \\ 0 & \frac{1}{b} \end{bmatrix} \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} = A^{-1}A.$$

Hence, A^{-1} is an inverse of A and belongs to H. □

2. Find all generators of the following groups.

2.1 It easy to see that $\langle 1 \rangle = \mathbb{Z}_{36}$ and $\circ(\mathbb{Z}_{36}) = 36$. If $\gcd(k, 36) = 1$, then $k = 1, 5, 7, 11, 13, 17, 19, 23, 25, 29, 31, 35$. Hence, all generators of $\mathbb{Z}_{36}$ are

$$\langle 1 \rangle, \langle 5 \rangle, \langle 7 \rangle, \langle 11 \rangle, \langle 13 \rangle, \langle 17 \rangle, \langle 19 \rangle, \langle 23 \rangle, \langle 25 \rangle, \langle 29 \rangle, \langle 31 \rangle, \langle 35 \rangle.$$

2.2 $\mathbb{Z}_{25}^{\times} = \{1, 2, 3, 4, 6, 7, 8, 9, 11, 12, 13, 14, 16, 17, 18, 19, 21, 22, 23, 24\}$. Then $\circ(\mathbb{Z}_{25}^{\times}) = 20$. Since

$$\begin{aligned} \langle 2 \rangle &= \{2^0, 2^1, 2^2, 2^3, 2^4, 2^5, 2^6, 2^7, 2^8, 2^9, 2^{10}, 2^{11}, 2^{11}, 2^{12}, 2^{13}, 2^{14}, 2^{15}, 2^{16}, 2^{17}, 2^{18}, 2^{19}\} \\ &= \{1, 2, 4, 8, 16, 12, 14, 3, 6, 12, 24, 23, 21, 17, 9, 18, 11, 22, 19, 13\} = \mathbb{Z}_{25}^{\times}, \end{aligned}$$

2 is a generator of $\mathbb{Z}_{25}^{\times}$. If $\gcd(k, 20) = 1$, $k = 1, 3, 7, 9, 11, 13, 17, 19$. Hence, all generators of $\mathbb{Z}_{25}^{\times}$ are

$$\langle 2^1 \rangle, \langle 2^3 \rangle, \langle 2^7 \rangle, \langle 2^9 \rangle, \langle 2^{11} \rangle, \langle 2^{13} \rangle, \langle 2^{17} \rangle, \langle 2^{19} \rangle \quad \text{or} \quad \langle 2 \rangle, \langle 8 \rangle, \langle 3 \rangle, \langle 12 \rangle, \langle 23 \rangle, \langle 17 \rangle, \langle 22 \rangle, \langle 13 \rangle.$$

3. Find all subgroups of the following groups by Lagrange's theorem.

3.1 It easy to see that $\langle 1 \rangle = \mathbb{Z}_{18}$ and $\circ(\mathbb{Z}_{18}) = 18$. All divisors of 18 is 1, 2, 3, 6, 9 and 18. By Lagrance's theorem, all subgroups of $\mathbb{Z}_{18}$ are $\left\langle 1^{\frac{18}{1}} \right\rangle, \left\langle 1^{\frac{18}{2}} \right\rangle, \left\langle 1^{\frac{18}{3}} \right\rangle, \left\langle 1^{\frac{18}{6}} \right\rangle, \left\langle 1^{\frac{18}{9}} \right\rangle, \left\langle 1^{\frac{18}{18}} \right\rangle$. We obtain

$$\langle 0 \rangle, \langle 9 \rangle, \langle 6 \rangle, \langle 3 \rangle, \langle 2 \rangle, \langle 1 \rangle$$

3.2 By 2.2, $\langle 2 \rangle = \mathbb{Z}_{25}^{\times}$ and $\circ(\mathbb{Z}_{25}^{\times}) = 20$. All dibvisors of 20 are 1, 2, 4, 5, 10 and 20. By Lagrance's theorem, all subgroups of $\mathbb{Z}_{25}^{\times}$ are $\left\langle 2^{\frac{20}{1}} \right\rangle, \left\langle 2^{\frac{20}{2}} \right\rangle, \left\langle 2^{\frac{20}{4}} \right\rangle, \left\langle 2^{\frac{20}{5}} \right\rangle, \left\langle 2^{\frac{20}{10}} \right\rangle, \left\langle 2^{\frac{20}{20}} \right\rangle$. We obtain

$$\langle 1 \rangle, \langle 24 \rangle, \langle 7 \rangle, \langle 16 \rangle, \langle 4 \rangle, \langle 2 \rangle.$$

ANSWER QUIZ 2 : MAT2303 ABSTRACT (SEC2)

TOPIC Subgroups & Cyclic groups **SCORE** 10 points
QUIZ TIME Wed 14 Sep 2016, 6th Week, Semester 1/2016
TEACHER Thanatyod Jampawai, Ph.D., Faculty of Education,
Suan Sunandha Rajabhat University

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1. Let $H = \left\{ \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} : ab > 0 \right\}$. Prove that H is a subgroup of $GL_2(\mathbb{R})$.

   Proof. We first choose $a = b = 1$, $ab = 1 > 0$. So, $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ belongs to H.
   Next, we will show that H is closed. Let $A = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$ and $B = \begin{bmatrix} x & 0 \\ 0 & y \end{bmatrix}$ be elements in H. Then

   $$AB = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \begin{bmatrix} x & 0 \\ 0 & y \end{bmatrix} = \begin{bmatrix} ax & 0 \\ 0 & by \end{bmatrix}$$

   Since $A, B \in H$, $ab > 0$ and $xy > 0$. We conclude that $(ax)(by) = (ab)(xy) > 0$. Thus, $AB \in H$.
   Finally, let $A = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$ be in H. Then $ab > 0$. It follows that $a \neq 0$ and $b \neq 0$. Choose $A^{-1} = \begin{bmatrix} \frac{1}{a} & 0 \\ 0 & \frac{1}{b} \end{bmatrix}$.
   Since ab is positive, $\frac{1}{ab}$ is also positive. Then

   $$AA^{-1} \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \begin{bmatrix} \frac{1}{a} & 0 \\ 0 & \frac{1}{b} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{a} & 0 \\ 0 & \frac{1}{b} \end{bmatrix} \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} = A^{-1}A.$$

   Hence, A^{-1} is an inverse of A and belongs to H. □

2. Find all generators of the following groups.

   2.1 It easy to see that $\langle 1 \rangle = \mathbb{Z}_{48}$ and $\circ(\mathbb{Z}_{48}) = 48$. If $\gcd(k, 48) = 1$, then k= 1, 5, 7, 11, 13, 17, 19, 23, 25, 29, 31, 35, 37, 41, 43 47. Hence, all generators of $\mathbb{Z}_{48}$ are

   $$\langle 1 \rangle, \langle 5 \rangle, \langle 7 \rangle, \langle 11 \rangle, \langle 13 \rangle, \langle 17 \rangle, \langle 19 \rangle, \langle 23 \rangle, \langle 25 \rangle, \langle 29 \rangle, \langle 31 \rangle, \langle 35 \rangle, \langle 29 \rangle, \langle 31 \rangle, \langle 35 \rangle, \langle 37 \rangle, \langle 41 \rangle, \langle 43 \rangle, \langle 47 \rangle.$$

   2.2 $\mathbb{Z}_{25}^{\times} = \{1, 2, 3, 4, 6, 7, 8, 9, 11, 12, 13, 14, 16, 17, 18, 19, 21, 22, 23, 24\}$. Then $\circ(\mathbb{Z}_{25}^{\times}) = 20$. Since

   $$\begin{aligned} \langle 2 \rangle &= \{2^0, 2^1, 2^2, 2^3, 2^4, 2^5, 2^6, 2^7, 2^8, 2^9, 2^{10}, 2^{11}, 2^{11}, 2^{12}, 2^{13}, 2^{14}, 2^{15}, 2^{16}, 2^{17}, 2^{18}, 2^{19}\} \\ &= \{1, 2, 4, 8, 16, 12, 14, 3, 6, 12, 24, 23, 21, 17, 9, 18, 11, 22, 19, 13\} = \mathbb{Z}_{25}^{\times}, \end{aligned}$$

   2 is a generator of $\mathbb{Z}_{25}^{\times}$. If $\gcd(k, 20) = 1$, $k = 1, 3, 7, 9, 11, 13, 17, 19$. Hence, all generators of $\mathbb{Z}_{25}^{\times}$ are

   $$\langle 2^1 \rangle, \langle 2^3 \rangle, \langle 2^7 \rangle, \langle 2^9 \rangle, \langle 2^{11} \rangle, \langle 2^{13} \rangle, \langle 2^{17} \rangle, \langle 2^{19} \rangle \quad \text{or} \quad \langle 2 \rangle, \langle 8 \rangle, \langle 3 \rangle, \langle 7 \rangle, \langle 23 \rangle, \langle 17 \rangle, \langle 22 \rangle, \langle 13 \rangle.$$

3. Find all subgroups of the following groups by Lagrange's theorem.

   3.1 It easy to see that $\langle 1 \rangle = \mathbb{Z}_{18}$ and $\circ(\mathbb{Z}_{24}) = 24$. All divisors of 24 is 1, 2, 3, 4, 6, 8, 12 and 24. By Lagrance's theorem, all subgroups of $\mathbb{Z}_{18}$ are $\left\langle 1^{\frac{24}{1}} \right\rangle, \left\langle 1^{\frac{24}{2}} \right\rangle, \left\langle 1^{\frac{24}{3}} \right\rangle, \left\langle 1^{\frac{24}{4}} \right\rangle, \left\langle 1^{\frac{24}{6}} \right\rangle, \left\langle 1^{\frac{24}{8}} \right\rangle, \left\langle 1^{\frac{24}{12}} \right\rangle, \left\langle 1^{\frac{24}{24}} \right\rangle$. Then

   $$\langle 0 \rangle, \langle 12 \rangle, \langle 8 \rangle, \langle 6 \rangle, \langle 4 \rangle, \langle 3 \rangle, \langle 2 \rangle, \langle 1 \rangle$$

   3.2 By 2.2, $\langle 2 \rangle = \mathbb{Z}_{25}^{\times}$ and $\circ(\mathbb{Z}_{25}^{\times}) = 20$. All dibvisors of 20 are 1, 2, 4, 5, 10 and 20. By Lagrance's theorem, all subgroups of $\mathbb{Z}_{25}^{\times}$ are $\left\langle 2^{\frac{20}{1}} \right\rangle, \left\langle 2^{\frac{20}{2}} \right\rangle, \left\langle 2^{\frac{20}{4}} \right\rangle, \left\langle 2^{\frac{20}{5}} \right\rangle, \left\langle 2^{\frac{20}{10}} \right\rangle, \left\langle 2^{\frac{20}{20}} \right\rangle$. We obtain

   $$\langle 1 \rangle, \langle 24 \rangle, \langle 7 \rangle, \langle 16 \rangle, \langle 4 \rangle, \langle 2 \rangle.$$

QUIZ 2 : MAT2303 ABSTRACT ALGEBRA

TOPIC Quotient groups & Homomorphism **SCORE** 10 points
QUIZ TIME Wed 12 Oct 2016, 9th Week, Semester 1/2016
TEACHER Thanatyod Jampawai, Ph.D., Faculty of Education,
Suan Sunandha Rajabhat University
NAME... **ID**.................................... **SECTION**.......................

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1. **(3 points)** In quotient group,

   1.1 List all elements of $\mathbb{Z}_{12}/\langle 3\rangle$

   1.2 Find all inverses for each element in $\mathbb{Z}_{12}/\langle 3\rangle$

2. **(4 points)** Define a map $\varphi : (\mathbb{R}^+, \cdot) \to (\mathbb{R}, +)$ by $\varphi(x) = \ln x$

   2.1 Prove that φ is isomorphism

   2.2 Find $Ker(\varphi)$

3. **(3 points)** Define a map from $\mathbb{Z}_6 = \{0, 1, 2, 3, 4, 5\}$ to $\mathbb{Z}_2 \times \mathbb{Z}_3 = \{(0,0), (0,1), (0,2), (1,0), (1,1), (1,2)\}$ by

$$0 \mapsto (0,0),\ 1 \mapsto (1,1),\ 2 \mapsto (0,2),\ 3 \mapsto (1,0),\ 4 \mapsto (0,1)\ \text{ and }\ 5 \mapsto (1,2)$$

Show that the map is homomorphism by filling below tables and explain that $\mathbb{Z}_6 \cong \mathbb{Z}_2 \times \mathbb{Z}_3$.

+	0	1	2	3	4	5
0	0					
1		2				
2						
3		4				
4				1		
5						

$\mapsto$

map	0	1	2	3	4	5
0	(0,0)					
1		(0,2)				
2						
3		(0,1)				
4				(1,1)		
5						

map +	0 ↓ (0,0)	1 ↓ (1,1)	2 ↓ (0,2)	3 ↓ (1,0)	4 ↓ (0,1)	5 ↓ (1,2)
$0 \mapsto$ (0,0)	(0,0)					
$1 \mapsto$ (1,1)		(0,2)				
$2 \mapsto$ (0,2)						
$3 \mapsto$ (1,0)		(0,1)				
$4 \mapsto$ (0,1)				(1,1)		
$5 \mapsto$ (1,2)						

QUIZ 2 : MAT2303 ABSTRACT ALGEBRA

TOPIC Quotient groups & Homomorphism **SCORE** 10 points
QUIZ TIME Wed 12 Oct 2016, 9th Week, Semester 1/2016
TEACHER Thanatyod Jampawai, Ph.D., Faculty of Education,
Suan Sunandha Rajabhat University
NAME.. **ID**.................................... **SECTION**.......................

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1. **(3 points)** In quotient group,

   1.1 List all elements of $\mathbb{Z}_{15}/\langle 5\rangle$

   1.2 Find all inverses for each element in $\mathbb{Z}_{15}/\langle 5\rangle$

2. **(4 points)** Define a map $\varphi : (M_{22}(\mathbb{Z}), +) \to (\mathbb{Z}, +)$ by

$$\varphi(\begin{bmatrix} a & b \\ c & d \end{bmatrix}) = a + d$$

   2.1 Prove that φ is isomorphism

   2.2 Find $Ker(\varphi)$

3. **(3 points)** Define a map from $\mathbb{Z}_6 = \{0, 1, 2, 3, 4, 5\}$ to $\mathbb{Z}_2 \times \mathbb{Z}_3 = \{(0,0), (0,1), (0,2), (1,0), (1,1), (1,2)\}$ by

$$0 \mapsto (0,0),\ 1 \mapsto (1,1),\ 2 \mapsto (0,2),\ 3 \mapsto (1,0),\ 4 \mapsto (0,1)\ \text{ and }\ 5 \mapsto (1,2)$$

Show that the map is homomorphism by filling below tables and explain that $\mathbb{Z}_6 \cong \mathbb{Z}_2 \times \mathbb{Z}_3$.

+	0	1	2	3	4	5
0	0					
1		2				
2						
3		4				
4				1		
5						

$\mapsto$

map	0	1	2	3	4	5
0	(0,0)					
1		(0,2)				
2						
3		(0,1)				
4				(1,1)		
5						

map / +	0 $\downarrow$ (0,0)	1 $\downarrow$ (1,1)	2 $\downarrow$ (0,2)	3 $\downarrow$ (1,0)	4 $\downarrow$ (0,1)	5 $\downarrow$ (1,2)
$0 \mapsto$ (0,0)	(0,0)					
$1 \mapsto$ (1,1)		(0,2)				
$2 \mapsto$ (0,2)						
$3 \mapsto$ (1,0)		(0,1)				
$4 \mapsto$ (0,1)				(1,1)		
$5 \mapsto$ (1,2)						