QUIZ 1 : MAT2305 LINEAR ALGEBRA

TOPIC Linear equation system & Matrix **SCORE** 10 points
QUIZ TIME Mon 22 Jan 2018, 3th Week, Semester 2/2017
TEACHER Thanatyod Jampawai, Ph.D., Faculty of Education, Suan Sunandha Rajabhat University
NAME.. **ID**.................................... **SECTION**........................

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1. **(3 points)** Use Guass-Jardan elimination to solve linear equation systems

$$\begin{cases} x_1 + 3x_2 + 2x_3 + x_4 & = 1 \\ x_1 + 2x_2 + 2x_3 + x_4 & = 9 \\ 3x_1 + 7x_2 + 6x_3 + 3x_4 & = -5 \end{cases}$$

2. **(5 points)** Find the inverse of

$$A = \begin{bmatrix} 2 & 3 & 2 \\ 1 & 2 & 3 \\ 1 & 1 & -2 \end{bmatrix}$$

3. **(2 points)** Write linear system equation system in $A\boldsymbol{x} = \boldsymbol{b}$ and use inverse of A in 2 to solve the system.

$$\begin{cases} 2x_1 + 3x_2 + 2x_3 & = 1 \\ x_1 + 2x_2 + 3x_3 & = 2 \\ x_1 + x_2 - 2x_3 & = 3 \end{cases}$$

SOLUTIONS QUIZ 1
MAT2305 LINEAR ALGEBRA (SEC1)

TOPIC Linear equation system & Matrix **SCORE** 10 points
QUIZ TIME Mon 22 Jan 2018, 3th Week, Semester 2/2017
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1. **(3 points)** Use Guass-Jardan elimination to solve linear equation systems

$$\begin{cases} x_1 + 3x_2 + 2x_3 + x_4 & = 1 \\ x_1 + 2x_2 + 2x_3 + x_4 & = 9 \\ 3x_1 + 7x_2 + 6x_3 + 3x_4 & = -5 \end{cases}$$

Solution. Write out the linear equation system in augmented matrix to be

$$\left[\begin{array}{cccc|c} 1 & 3 & 2 & 1 & 1 \\ 1 & 2 & 2 & 1 & 9 \\ 3 & 7 & 6 & 3 & -5 \end{array}\right]$$

Then

$$\left[\begin{array}{cccc|c} 1 & 3 & 2 & 1 & 1 \\ 1 & 2 & 2 & 1 & 9 \\ 3 & 7 & 6 & 3 & -5 \end{array}\right] \xrightarrow[R_2-R_1]{R_3-3R_1} \left[\begin{array}{cccc|c} 1 & 3 & 2 & 1 & 1 \\ 0 & -1 & 0 & 0 & 8 \\ 0 & -2 & 0 & 0 & -8 \end{array}\right] \xrightarrow{R_3-2R_2} \left[\begin{array}{cccc|c} 1 & 3 & 2 & 1 & 1 \\ 0 & -1 & 0 & 0 & 8 \\ 0 & 0 & 0 & 0 & -24 \end{array}\right]$$

Thus,

$$\begin{array}{ccccccccc} x_1 & + & 3x_2 & + & 2x_3 & + & x_4 & = & 1 \\ & - & x_2 & & & & & = & 8 \\ & & & & & & 0 & = & -24 \end{array}$$

Hence, the linear system has no solution (inconsistent). #

2. **(5 points)** Find the inverse of

$$A = \begin{bmatrix} 2 & 3 & 2 \\ 1 & 2 & 3 \\ 1 & 1 & -2 \end{bmatrix}$$

Solution.

$$\left[\begin{array}{ccc|ccc} 2 & 3 & 2 & 1 & 0 & 0 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 1 & 1 & -2 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_{12}} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 0 & 1 & 0 \\ 2 & 3 & 2 & 1 & 0 & 0 \\ 1 & 1 & -2 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow[R_3-R1]{R_2-2R_1} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 0 & 1 & 0 \\ 0 & -1 & -4 & 1 & -2 & 0 \\ 0 & -1 & -5 & 0 & -1 & 1 \end{array}\right]$$

$$\xrightarrow{(-1)R_2} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 0 & 1 & 0 \\ 0 & 1 & 4 & -1 & 2 & 0 \\ 0 & -1 & -5 & 0 & -1 & 1 \end{array}\right]$$

$$\xrightarrow[R_3+R_2]{R_1-2R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & -5 & 2 & -3 & 0 \\ 0 & 1 & 4 & -1 & 2 & 0 \\ 0 & 0 & -1 & -1 & 1 & 1 \end{array}\right]$$

$$\xrightarrow{(-1)R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & -5 & 2 & -3 & 0 \\ 0 & 1 & 4 & -1 & 2 & 0 \\ 0 & 0 & 1 & 1 & -1 & -1 \end{array}\right]$$

$$\xrightarrow[R_2-4R_3]{R_1-5R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 7 & -8 & -5 \\ 0 & 1 & 0 & -5 & 6 & 4 \\ 0 & 0 & 1 & 1 & -1 & -1 \end{array}\right]$$

Thus,

$$A^{-1} = \begin{bmatrix} 7 & -8 & -5 \\ -5 & 6 & 4 \\ 1 & -1 & -1 \end{bmatrix} \quad \#$$

3. **(2 points)** Write linear system equation system in $A\boldsymbol{x} = \boldsymbol{b}$ and use inverse of A in 2 to solve the system.

$$\begin{cases} 2x_1 + 3x_2 + 2x_3 & = 1 \\ x_1 + 2x_2 + 3x_3 & = 2 \\ x_1 + x_2 - 2x_3 & = 3 \end{cases}$$

Solution. We write linear system equation system in $A\boldsymbol{x} = \boldsymbol{b}$, i.e.,

$$\begin{bmatrix} 2 & 3 & 2 \\ 1 & 2 & 3 \\ 1 & 1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Taking A^{-1} to two left sides and By 2, we obtain

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 & 3 & 2 \\ 1 & 2 & 3 \\ 1 & 1 & -2 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 7 & -8 & -5 \\ -5 & 6 & 4 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 7-16-15 \\ -5+12+12 \\ 1-2-3 \end{bmatrix} = \begin{bmatrix} -24 \\ 19 \\ -4 \end{bmatrix}$$

Therefore, $x_1 = -24, x_2 = 19, x_3 = -4$ is the solution. #

QUIZ 1 : MAT2305 LINEAR ALGEBRA

TOPIC Linear equation system & Matrix **SCORE** 10 points
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Suan Sunandha Rajabhat University
NAME.. **ID**.................................... **SECTION**.......................

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1. **(3 points)** Use Guass-Jardan elimination to solve linear equation systems

$$\begin{cases} x_1 + 2x_2 + 3x_3 + 4x_4 & = 1 \\ x_1 + x_2 + 2x_3 + 2x_4 & = 0 \\ 3x_1 + 4x_2 + 7x_3 + 8x_4 & = 1 \end{cases}$$

2. **(5 points)** Find the inverse of

$$A = \begin{bmatrix} 2 & 3 & 6 \\ 1 & 3 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

3. **(2 points)** Write linear system equation system in $A\boldsymbol{x} = \boldsymbol{b}$ and use inverse of A in 2 to solve the system.

$$\begin{cases} 2x_1 + 3x_2 + 6x_3 & = 1 \\ x_1 + 3x_2 + 2x_3 & = 2 \\ x_1 + 2x_2 + 3x_3 & = 3 \end{cases}$$

SOLUTIONS QUIZ 1
MAT2305 LINEAR ALGEBRA (SEC2)

TOPIC Linear equation system & Matrix **SCORE** 10 points
QUIZ TIME Mon 22 Jan 2018, 3th Week, Semester 2/2017
TEACHER Thanatyod Jampawai, Ph.D., Faculty of Education,
Suan Sunandha Rajabhat University
NAME... **ID**................................... **SECTION**......................

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1. **(3 points)** Use Guass-Jardan elimination to solve linear equation systems

$$\begin{cases} x_1 + 2x_2 + 3x_3 + 4x_4 &= 1 \\ x_1 + x_2 + 2x_3 + 2x_4 &= 0 \\ 3x_1 + 4x_2 + 7x_3 + 8x_4 &= 1 \end{cases}$$

Solution. Write out the linear equation system in augmented matrix to be

$$\left[\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 1 \\ 1 & 1 & 2 & 2 & 0 \\ 3 & 4 & 7 & 8 & 1 \end{array}\right]$$

Then

$$\left[\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 1 \\ 1 & 1 & 2 & 2 & 0 \\ 3 & 4 & 7 & 8 & 1 \end{array}\right] \xrightarrow[R_3-3R_1]{R_2-R_1} \left[\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 1 \\ 0 & -1 & -1 & -2 & -1 \\ 0 & -2 & -2 & -4 & -2 \end{array}\right] \xrightarrow[R_3-2R_2]{R_1+2R_2} \left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & -1 \\ 0 & -1 & -1 & -2 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Thus,

$$\begin{array}{ccccccccc} x_1 & & & + & x_3 & & & = & -1 \\ & - & x_2 & - & x_3 & - & 2x_4 & = & -1 \end{array}$$

Let t and s be a free variable. Setting $x_3 = t$ and $x_4 = s$. Then

$$x_1 = -1 - t \quad \text{and} \quad x_2 = 1 - t - 2s$$

Hence, $x_1 = -1 - t, x_2 = 1 - t - 2s, x_3 = t, x_4 = s$ is the solution. #

2. **(5 points)** Find the inverse of

$$A = \begin{bmatrix} 2 & 3 & 6 \\ 1 & 3 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

Solution.

$$\left[\begin{array}{ccc|ccc} 2 & 3 & 6 & 1 & 0 & 0 \\ 1 & 3 & 2 & 0 & 1 & 0 \\ 1 & 2 & 3 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_{12}} \left[\begin{array}{ccc|ccc} 1 & 3 & 2 & 0 & 1 & 0 \\ 2 & 3 & 6 & 1 & 0 & 0 \\ 1 & 2 & 3 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow[R_3-R1]{R_2-2R_1} \left[\begin{array}{ccc|ccc} 1 & 3 & 2 & 0 & 1 & 0 \\ 0 & -3 & 2 & 1 & -2 & 0 \\ 0 & -1 & 1 & 0 & -1 & 1 \end{array}\right]$$

$$\xrightarrow{R_{23}} \left[\begin{array}{ccc|ccc} 1 & 3 & 2 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & -1 & 1 \\ 0 & -3 & 2 & 1 & -2 & 0 \end{array}\right]$$

$$\xrightarrow{(-1)R_2} \left[\begin{array}{ccc|ccc} 1 & 3 & 2 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 & 1 & -1 \\ 0 & -3 & 2 & 1 & -2 & 0 \end{array}\right]$$

$$\xrightarrow[R_3+3R_2]{R_1-3R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 5 & 0 & -2 & 3 \\ 0 & 1 & -1 & 0 & 1 & -1 \\ 0 & 0 & -1 & 1 & 1 & -3 \end{array}\right]$$

$$\xrightarrow{(-1)R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 5 & 0 & -2 & 3 \\ 0 & 1 & -1 & 0 & 1 & -1 \\ 0 & 0 & 1 & -1 & -1 & 3 \end{array}\right]$$

$$\xrightarrow[R_2+R_3]{R_1-5R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 5 & 3 & -12 \\ 0 & 1 & 0 & -1 & 0 & 2 \\ 0 & 0 & 1 & -1 & -1 & 3 \end{array}\right]$$

Thus,

$$A^{-1} = \begin{bmatrix} 5 & 3 & -12 \\ -1 & 0 & 2 \\ -1 & -1 & 3 \end{bmatrix} \quad \#$$

3. **(2 points)** Write linear system equation system in $A\boldsymbol{x} = \boldsymbol{b}$ and use inverse of A in 2 to solve the system.

$$\begin{cases} 2x_1 + 3x_2 + 6x_3 &= 1 \\ x_1 + 3x_2 + 2x_3 &= 2 \\ x_1 + 2x_2 + 3x_3 &= 3 \end{cases}$$

Solution. We write linear system equation system in $A\boldsymbol{x} = \boldsymbol{b}$, i.e.,

$$\begin{bmatrix} 2 & 3 & 6 \\ 1 & 3 & 2 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Taking A^{-1} to two left sides and By 2, we obtain

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 & 3 & 6 \\ 1 & 3 & 2 \\ 1 & 2 & 3 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 5 & 3 & -12 \\ -1 & 0 & 2 \\ -1 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 5+6-36 \\ -1+0+6 \\ -1-2+9 \end{bmatrix} = \begin{bmatrix} -25 \\ 5 \\ 6 \end{bmatrix}$$

Therefore, $x_1 = -25, x_2 = 5, x_3 = 6$ is the solution. #

QUIZ 2 : MAT2305 LINEAR ALGEBRA

TOPIC LU-decomposition & Determinance **SCORE** 10 points
QUIZ TIME Mon 5 Feb 2018, 5th Week, Semester 2/2017
TEACHER Thanatyod Jampawai, Ph.D., Faculty of Education,
Suan Sunandha Rajabhat University

NAME.. **ID**.................................... **SECTION**.......................

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1. **(4 points)** Solve the linear system by LU-decomposition

$$\begin{cases} x_1 + x_2 + 3x_3 & = 12 \\ x_1 + 2x_2 + 2x_3 & = 11 \\ x_1 - x_2 + x_3 & = 2 \end{cases}$$

2. **(3 points)** Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ and $B = \begin{bmatrix} a_{21} - 3a_{11} & a_{22} - 3a_{12} \\ 2a_{11} & 2a_{12} \end{bmatrix}$. If $\det(A) = 2$, find

   2.1 $\det(3AB^2)$ 2.2 $\det(4A^T(2B^2)^{-1})$ 2.3 $\det(A + B)$

3. **(3 points)** The linear system $A\boldsymbol{x} = \boldsymbol{b}$ with variables x_1, x_2, x_3, x_4 has

$$x_1 = \frac{\begin{vmatrix} 1 & 0 & 1 & 0 \\ 1 & 2 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 1 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}}$$

   Use **Cramer's rule** to find x_2, x_3, x_4

SOLUTIONS QUIZ 2
MAT2305 LINEAR ALGEBRA (SEC1)

TOPIC	LU-decomposition & Determinance	**SCORE**	10 points
QUIZ TIME	Mon 5 Feb 2018, 5th Week, Semester 2/2017		
TEACHER	Thanatyod Jampawai, Ph.D., Faculty of Education, Suan Sunandha Rajabhat University		

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1. **(4 points)** Solve the linear system by LU-decomposition

$$\begin{cases} x_1 + x_2 + 3x_3 &= 12 \\ x_1 + 2x_2 + 2x_3 &= 11 \\ x_1 - x_2 + x_3 &= 2 \end{cases}$$

Solution.

$$A\boldsymbol{x} = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 2 & 2 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 11 \\ 2 \end{bmatrix} = \boldsymbol{b}$$

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 2 & 2 \\ 1 & -1 & 1 \end{bmatrix} \xrightarrow[(-1)R_1+R_2]{(-1)R_1+R_3} \begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & -1 \\ 0 & -2 & -2 \end{bmatrix} \xrightarrow{(+2)R_2+R_3} \begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & -1 \\ 0 & 0 & -4 \end{bmatrix} = U$$

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow[(+1)R_1+R_2]{(+1)R_1+R_3} \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \xrightarrow{(-2)R_2+R_3} \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix} = L$$

$$A\boldsymbol{x} = LU\boldsymbol{x} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & -1 \\ 0 & 0 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 11 \\ 2 \end{bmatrix}$$

Setting $\boldsymbol{y} = U\boldsymbol{x}$, that is

$$U\boldsymbol{x} = \begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & -1 \\ 0 & 0 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \boldsymbol{y}$$

Then

$$L\boldsymbol{y} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 11 \\ 2 \end{bmatrix} = \boldsymbol{b}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 11 \\ 2 \end{bmatrix} \quad \leftrightarrow \quad \begin{array}{rcrcrcr} y_1 & & & & & = & 12 \\ y_1 & + & y_2 & & & = & 11 \\ y_1 & - & 2y_2 & + & y_3 & = & 2 \end{array}$$

$$\begin{array}{rccl} & \rightarrow & \therefore & y_1 = 12 \\ y_1 + y_2 = 11 & \rightarrow & \therefore & y_2 = 11 - 12 = -1 \\ y_1 - 2y_2 + y_3 = 2 & \rightarrow & \therefore & y_3 = 2 - 12 + 2(-1) = -12 \end{array}$$

Since $\boldsymbol{y} = U\boldsymbol{x}$,

$$U\boldsymbol{x} = \begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & -1 \\ 0 & 0 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 12 \\ -1 \\ -12 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & -1 \\ 0 & 0 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 12 \\ -1 \\ -12 \end{bmatrix} \quad \leftrightarrow \quad \begin{array}{rcrcrcr} x_1 & + & x_2 & - & x_3 & = & 0 \\ & - & 3x_2 & + & x_3 & = & -3 \\ & & & & x_3 & = & 5 \end{array}$$

$$\begin{aligned} -4x_3 = -12 &\quad \rightarrow \quad \therefore \quad x_3 = 3 \\ x_2 - x_3 = -1 &\quad \rightarrow \quad \therefore \quad x_2 = -1 + 3 = 2 \\ x_1 + x_2 + 3x_3 = 12 &\quad \rightarrow \quad \therefore \quad x_1 = 12 - 2 - 3(3) = 1 \end{aligned}$$

Verify your answer

$$\begin{cases} x_1 + x_2 + 3x_3 &= 12 \\ x_1 + 2x_2 + 2x_3 &= 11 \\ x_1 - x_2 + x_3 &= 2 \end{cases} \quad \rightarrow \quad \begin{cases} 1 + 2 + 3(3) &= 12 \\ 1 + 2(2) + 2(3) &= 11 \\ 1 - 2 + 3 &= 2 \end{cases}$$

Thus, $x_1 = 1, x_2 = 2, x_3 = 3$ is the solution. #

2. **(3 points)** Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ and $B = \begin{bmatrix} a_{21} - 3a_{11} & a_{22} - 3a_{12} \\ 2a_{11} & 2a_{12} \end{bmatrix}$. If $\det(A) = 2$, find

Solution. Consider,

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \xrightarrow{R_{12}} \begin{bmatrix} a_{21} & a_{22} \\ a_{11} & a_{12} \end{bmatrix} \xrightarrow{R_1 - 3R_2} \begin{bmatrix} a_{21} - 3a_{11} & a_{22} - 3a_{12} \\ a_{11} & a_{12} \end{bmatrix} \xrightarrow{2R_2} \begin{bmatrix} a_{21} - 3a_{11} & a_{22} - 3a_{12} \\ 2a_{11} & 2a_{12} \end{bmatrix} = B$$

Thus, $\det(B) = -2(2) = -4$

2.1 $\det(3AB^2) = 3^2 \det(A)[\det(B)]^2 = 9 \cdot 2 \cdot (-4)^2 = 288$ #

2.2 $\det(4A^T(2B^2)^{-1}) = 4^2 \det(A^T)\frac{1}{\det(2B^2)} = 16 \det(A)\frac{1}{2^2 \det B^2} = 4(2)\frac{1}{(-4)^2} = \frac{1}{2}$ #

2.3 $\det(A + B)$

$$\begin{aligned} \det(A + B) &= \begin{vmatrix} a_{21} - 2a_{11} & a_{22} - 2a_{12} \\ 2a_{11} + a_{21} & 2a_{12} + a_{22} \end{vmatrix} = \begin{vmatrix} 2a_{21} & 2a_{22} \\ a_{11} + a_{21} & a_{12} + a_{22} \end{vmatrix}_{R_1 + R_2} \\ &= 2\begin{vmatrix} a_{21} & a_{22} \\ a_{11} + a_{21} & a_{12} + a_{22} \end{vmatrix} = 2\begin{vmatrix} a_{21} & a_{22} \\ a_{11} & a_{12} \end{vmatrix}_{R_2 - R_1} \\ &= -2\begin{vmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{vmatrix}_{R_{12}} = (-2)\det(A) = (-2)2 = -4 \quad \# \end{aligned}$$

3. **(3 points)** The linear system $A\boldsymbol{x} = \boldsymbol{b}$ with variables x_1, x_2, x_3, x_4 has

$$x_1 = \frac{\begin{vmatrix} 1 & 0 & 1 & 0 \\ 1 & 2 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 1 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}}$$

Use **Cramer's rule** to find x_2, x_3, x_4

Solution. Write the linear system,

$$A\boldsymbol{x} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \boldsymbol{b}$$

Hence,

$$x_2 = \frac{\begin{vmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 1 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}} = \frac{1}{2}$$

$$x_3 = \frac{\begin{vmatrix} 1 & 0 & 1 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 1 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}} = \frac{0}{2} = 0$$

$$x_4 = \frac{\begin{vmatrix} 1 & 0 & 1 & 1 \\ 0 & 2 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 1 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}} = \frac{0}{2} = 0$$

Therefore, $x_2 = \frac{1}{2}, x_3 = 0, x_4 = 0 \quad \#$

QUIZ 2 : MAT2305 LINEAR ALGEBRA

TOPIC LU-decomposition & Determinance **SCORE** 10 points
QUIZ TIME Mon 5 Feb 2018, 5th Week, Semester 2/2017
TEACHER Thanatyod Jampawai, Ph.D., Faculty of Education,
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1. **(4 points)** Solve the linear system by LU-decomposition

$$\begin{cases} x_1 + 2x_2 + 3x_3 &= 14 \\ x_1 + x_2 + 3x_3 &= 12 \\ x_1 - x_2 + x_3 &= 2 \end{cases}$$

2. **(3 points)** Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ and $B = \begin{bmatrix} 3a_{11} & 3a_{12} \\ 2a_{21} + 5a_{11} & 2a_{22} + 5a_{12} \end{bmatrix}$. If $\det(B) = 30$, find

   2.1 $\det(3AB^2)$ 2.2 $\det(4A^T(2B^2)^{-1})$ 2.3 $\det(A + B)$

3. **(3 points)** Apply **Cramer's rule** to solve the linear system $A\boldsymbol{x} = \boldsymbol{b}$ with variables x_1, x_2, x_3, x_4 when

$$A_1 = \begin{vmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 1 \end{vmatrix} \quad \text{and} \quad A_2 = \begin{vmatrix} 2 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

   Find x_2, x_3, x_4

SOLUTIONS QUIZ 2
MAT2305 LINEAR ALGEBRA (SEC2)

TOPIC	LU-decomposition & Determinance	**SCORE**	10 points
QUIZ TIME	Mon 5 Feb 2018, 5th Week, Semester 2/2017		
TEACHER	Thanatyod Jampawai, Ph.D., Faculty of Education, Suan Sunandha Rajabhat University		

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1. **(4 points)** Solve the linear system by LU-decomposition

$$\begin{cases} x_1 + 2x_2 + 3x_3 &= 14 \\ x_1 + x_2 + 3x_3 &= 12 \\ x_1 - x_2 + x_3 &= 2 \end{cases}$$

Solution.

$$A\boldsymbol{x} = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 3 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 14 \\ 12 \\ 2 \end{bmatrix} = \boldsymbol{b}$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 3 \\ 1 & -1 & 1 \end{bmatrix} \xrightarrow[(-1)R_1+R_2]{(-1)R_1+R_3} \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 0 \\ 0 & -3 & -2 \end{bmatrix} \xrightarrow{(-3)R_2+R_3} \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix} = U$$

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow[(+1)R_1+R_2]{(+1)R_1+R_3} \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \xrightarrow{(+3)R_2+R_3} \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 3 & 1 \end{bmatrix} = L$$

$$A\boldsymbol{x} = LU\boldsymbol{x} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 14 \\ 12 \\ 2 \end{bmatrix}$$

Setting $\boldsymbol{y} = U\boldsymbol{x}$, that is

$$U\boldsymbol{x} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \boldsymbol{y}$$

Then

$$L\boldsymbol{y} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 3 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 14 \\ 12 \\ 2 \end{bmatrix} = \boldsymbol{b}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 3 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 14 \\ 12 \\ 2 \end{bmatrix} \quad \leftrightarrow \quad \begin{array}{lllllll} y_1 & & & & & = & 14 \\ y_1 & + & y_2 & & & = & 12 \\ y_1 & + & 3y_2 & + & y_3 & = & 2 \end{array}$$

$$\begin{aligned} & \rightarrow \quad \therefore \quad y_1 = 14 \\ y_1 + y_2 = 12 \quad & \rightarrow \quad \therefore \quad y_2 = 12 - 14 = -2 \\ y_1 + 3y_2 + y_3 = 2 \quad & \rightarrow \quad \therefore \quad y_3 = 2 - 14 - 3(-2) = -6 \end{aligned}$$

Since $\boldsymbol{y} = U\boldsymbol{x}$,

$$U\boldsymbol{x} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 14 \\ -2 \\ -6 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 14 \\ -2 \\ -6 \end{bmatrix} \quad \leftrightarrow \quad \begin{array}{rcrcrcr} x_1 & + & 2x_2 & + & 3x_3 & = & 14 \\ & - & x_2 & & & = & -2 \\ & & & - & 2x_3 & = & -6 \end{array}$$

$$\begin{aligned} -2x_3 = -6 \quad &\rightarrow \quad \therefore \quad x_3 = 3 \\ -x_2 = -2 \quad &\rightarrow \quad \therefore \quad x_2 = 2 \\ x_1 + 2x_2 + 3x_3 = 14 \quad &\rightarrow \quad \therefore \quad x_1 = 14 - 2(2) - 3(3) = 1 \end{aligned}$$

Verify your answer

$$\begin{cases} x_1 + 2x_2 + 3x_3 & = 14 \\ x_1 + x_2 + 3x_3 & = 12 \\ x_1 - x_2 + x_3 & = 2 \end{cases} \quad \rightarrow \quad \begin{cases} 1 + 2(2) + 3(3) & = 14 \\ 1 + 2 + 3(3) & = 12 \\ 1 - 2 + 3 & = 2 \end{cases}$$

Thus, $x_1 = 1, x_2 = 2, x_3 = 3$ is the solution. #

2. **(3 points)** Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ and $B = \begin{bmatrix} 3a_{11} & 3a_{12} \\ 2a_{21} + 5a_{11} & 2a_{22} + 5a_{12} \end{bmatrix}$. If $\det(B) = 30$, find

Solution. Consider,

$$\begin{aligned} 30 = \det(B) &= \begin{vmatrix} 3a_{11} & 3a_{12} \\ 2a_{21} + 5a_{11} & 2a_{22} + 5a_{12} \end{vmatrix} = 3 \begin{vmatrix} a_{11} & a_{12} \\ 2a_{21} + 5a_{11} & 2a_{22} + 5a_{12} \end{vmatrix} \\ &= 3 \begin{bmatrix} a_{11} & a_{12} \\ 2a_{21} & 2a_{22} \end{bmatrix}_{R_2 - 5R_1} = 3(2) \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \\ &= 6 \det(A) \end{aligned}$$

Thus, $\det(A) = 5$

2.1 $\det(3AB^2) = 3^2 \det(A)[\det(B)]^2 = 9 \cdot 5 \cdot (30)^2 = 40500$ #

2.2 $\det(4A^T(2B^2)^{-1}) = 4^2 \det(A^T) \frac{1}{\det(2B^2)} = 16 \det(A) \frac{1}{2^2 \det B^2} = 4(5) \frac{1}{(30)^2} = \frac{1}{45}$ #

2.3 $\det(A + B)$

$$\begin{aligned} \det(A + B) &= \begin{vmatrix} 4a_{11} & 4a_{12} \\ 3a_{11} + 5a_{21} & 3a_{12} + 5a_{22} \end{vmatrix} = 4 \begin{vmatrix} a_{11} & a_{12} \\ 3a_{11} + 5a_{21} & 3a_{12} + 5a_{22} \end{vmatrix} \\ &= 4 \begin{vmatrix} a_{11} & a_{12} \\ 3a_{11} & 3a_{12} \end{vmatrix}_{R_2 - 5R_1} = 4(3) \begin{vmatrix} a_{11} & a_{12} \\ a_{11} & a_{12} \end{vmatrix} \\ &= 12 \det(A) = 12(5) = 60 \quad \# \end{aligned}$$

3. **(3 points)** Apply **Cramer's rule** to solve the linear system $A\boldsymbol{x} = \boldsymbol{b}$ with variables x_1, x_2, x_3, x_4 when

$$A_1 = \begin{vmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 1 \end{vmatrix} \quad \text{and} \quad A_2 = \begin{vmatrix} 2 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

Find x_2, x_3, x_4

Solution. From A_1 and A_2, we write the linear system,

$$A\boldsymbol{x} = \begin{bmatrix} 2 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \boldsymbol{b}$$

Hence,

$$x_2 = \frac{\begin{vmatrix} 2 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 1 \end{vmatrix}}{\begin{vmatrix} 2 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 1 \end{vmatrix}} = \frac{4}{4} = 1$$

$$x_3 = \frac{\begin{vmatrix} 2 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{vmatrix}}{\begin{vmatrix} 2 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 1 \end{vmatrix}} = \frac{0}{4} = 0$$

$$x_4 = \frac{\begin{vmatrix} 2 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix}}{\begin{vmatrix} 2 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 1 \end{vmatrix}} = \frac{0}{4} = 0$$

Therefore, $x_2 = 1, x_3 = 0, x_4 = 0$ #