Assignment 10
MAC3309 Mathematical Analysis

Topic	Reimann sum and the Fundametal Theorem of Calculus	**Score**	10 marks
Time	$12th$ Week		
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1. Le $f(x) = 2x^2 + 1$ where $x \in [0,1]$ and $P = \left\{ \frac{j}{n} : j = 0, 1, ..., n \right\}$ be a partition of $[0,1]$. Show that if $f(t_i)$ is choosen by the **right end point** and **left end point** in each subinterval, then two $I(f)$, depend on two methods, are **NOT** different.

2. Le $f(x) = 2x^2 + 1$ where $x \in [0,1]$ and $P = \left\{ \frac{j}{2^n} : j = 0, 1, ..., 2^n \right\}$ be a partition of $[0,1]$. Show that if $f(t_i)$ is choosen by the **right end point** and **left end point** in each subinterval, then two $I(f)$, depend on two methods, are **NOT** different.

3. Suppose that f is integrable on $[a,b]$ and $\alpha \in \mathbb{R}$. Prove that αf is also integrable on $[a,b]$ and
$$\int_a^b \alpha f(x)\, dx = \alpha \int_a^b f(x)\, dx.$$

4. Use the Reimann sum to prove that
$$\int_a^b \alpha \, dx = \alpha(b-a).$$

5. Use the chain rule and the first fundamental of calculus to prove that
$$\frac{d}{dx} \int_a^{u(x)} f(t)\, dt = f(u(x)) \cdot u'(x),$$
where a is a constant.

6. Use the first fundamental of calculus to find $F(0)$ and $F'(0)$. Define
$$F(x) = \int_{e^{-x}}^{e^x} \frac{1 + t \cdot f(t)}{1 + f(t)}\, dt \quad \text{where } x \geq 0.$$

7. Assume that f is differentiable on $(0,1)$ and integrable on $[0,1]$. Show that
$$\int_0^1 x f'(x) + f(x)\, dx = f(1).$$

8. Let $u(x)$ be a real function on $[0,1]$ and be diferentiable on $(0,1)$.
Assume that $u(x) \neq 0$ for all $x \in [0,1]$ and $u(0) = u(1)$. Show that
$$\int_0^1 \frac{u'(x)}{u(x)}\, dx = 0.$$