Assignment 11
MAC3309 Mathematical Analysis

Topic	Integration by part and Infinite series	**Score**	10 marks
Time	$13th$ Week		
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1. Let $f(x) = \int_0^{x^2} \sec^2(t^2)dt$. Use **integration by part** to show that

$$2\int_0^1 \sec^2(x^2)\,dx - 4\int_0^1 xf(x)\,dx = \tan 1.$$

2. Let $f : [-a, a] \to \mathbb{R}$ where $a > 0$. Suppose that $f(-x) = f(x)$ for all $x \in [-a, a]$. Show that

$$\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx.$$

   Hint: By dividing the integral into two parts and changing of variable.

3. Let $\int_{-1}^{0} f(t)\,dt = 2022$. Estimate the integral

$$\int_0^1 \frac{f\left(\frac{x-1}{x+1}\right)}{(x+1)^2}\,dx$$

4. Show that $\sum_{k=1}^{\infty} \ln\left(\frac{k(k+2)}{(k+1)^2}\right)$ converges and find its value.

5. Use Telescoping Seires to show that Guass' formula : $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$.

6. Use Telescoping Seires to show that

$$\sum_{k=1}^{\infty} \frac{k-1}{2^k}$$

   converges and find its values.

7. Find all $x \in \mathbb{R}$ for which

$$\sum_{k=1}^{\infty} 3(x^k - x^{k-1})(x^k + x^{k-1})$$

   converges. For each such x, find the value of this series.

8. Let π be a Pi constant. Show that

$$\sum_{k=1}^{\infty} \frac{1}{\pi^{k^2}}\left[1 - \frac{\pi^{2k}}{\pi} + \left(\frac{\pi^k}{\pi}\right)^k\right]$$

   converges and find its value.