



Assignment 12

MAC3309 Mathematical Analysis

Topic Test of Series **Score** 10 marks
Time 14th Week
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1. Determine whether the following series are convergent.

(a) $\sum_{k=1}^{\infty} \frac{\sqrt[k]{k}}{k}$

(b) $\sum_{k=1}^{\infty} \left(1 + \frac{1}{k}\right) k^{-\pi}$

2. Find all $p \in \mathbb{R}$ such that the following series are convergent.

(a) $\sum_{k=1}^{\infty} \frac{\ln k}{k^p}$ **Hint:** Use the Integral Test.

(b) $\sum_{k=2}^{\infty} \frac{1}{k(\ln k)^p}$ **Hint:** Use the Integral Test.

3. Prove that

$$\text{if } \sum_{k=1}^{\infty} |a_k| \text{ converges, then } \sum_{k=1}^{\infty} \frac{|a_k|}{k^p} \text{ converges for all } p > 0.$$

Hint: Use The Limit Comparison Test.

4. Use the **Limit Comparison Test** to show that

$$\sum_{k=1}^{\infty} \arctan\left(\frac{1}{k^p}\right) \text{ converges if } p > 1.$$