Assignment 13
MAC3309 Mathematical Analysis

Topic	Absolute convergent and Alternating series	**Score**	10 marks
Time	15*th* Week		
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1. Use the **Root Test** to find all of $x \in \mathbb{R}$ such that
$$\sum_{k=1}^{\infty}\left(\frac{(kx+1)^2}{k^2+1}\right)^k \qquad \text{converges.}$$

2. Use the **Ratio Test** to find all of $x \in \mathbb{R}$ such that Bessel function of first order $J_1(x)$ **converges** where
$$J_1(x) = \sum_{k=0}^{\infty}\frac{(-1)^k x^{2k+1}}{k!(k+1)!2^{2k+1}}.$$

3. Dethermine whether the following series are absolutely convergent or NOT.
   (a) $\displaystyle\sum_{k=1}^{\infty}\left(\frac{k+1}{k+2}\right)^{k^2}$
   (b) $\displaystyle\sum_{k=1}^{\infty}\frac{(1+(-1)^k)^k}{e^k}$

4. Dethermine whether the following series are conditionally convergent or NOT.
   (a) $\displaystyle\sum_{k=1}^{\infty}\frac{(-1)^k k^2}{2^k}$
   (b) $\displaystyle\sum_{k=1}^{\infty}\frac{(-1)^k \sin k}{k!}$

5. For each the following, find all values of $p \in \mathbb{R}$ for which the given series converges absolutely.
   (a) $\displaystyle\sum_{k=1}^{\infty}\frac{k^p}{p^k}$
   (b) $\displaystyle\sum_{k=1}^{\infty}\frac{2^{kp}k!}{k^k}$

6. Assume that $\displaystyle\sum_{k=1}^{\infty} a_k$ coverges absolutely. Use **Cauchy Criterion** to prove that
$$\sum_{k=1}^{\infty}\frac{a_k}{k} \qquad \text{coverges absolutely.}$$

7. Prove that
$$\sum_{k=1}^{\infty}(-1)^k \arctan\left(\frac{1}{k}\right)$$
is conditionally convergent.
Hint: Use Alternating Series Test and Limit Comparision Test.