Math SSRU

Assignment 1
MAC3309 Mathematical Analysis

Topic Ordered field axiom & Well-Ordering Principle **Score** 10 marks
Time $1st$ Week
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1. Define $(\sqrt{a})^2 = a$ for all $a \geq 0$. Prove that
$$\sqrt{x^2} = |x| \quad \text{for all } x \in \mathbb{R}.$$
2. Let a and b be real numbers. Prove that
$$\text{if} \quad 0 < a < b, \quad \text{then} \quad \sqrt{a} < \sqrt{b}.$$
3. Let $x \in \mathbb{R}$. Prove that
$$-1 \leq x \leq 2 \quad \text{implies} \quad |x^2 + x - 2| \leq 4|x - 1|.$$
4. Let x and y be two distinct real numbers. Prove that
$$\frac{x+y}{2} \text{ lies between } x \text{ and } y.$$
5. Let a and b be positive real numbers. Prove that
$$\sqrt{ab} \leq \frac{a+b}{2}.$$
6. Let a and b be positive real numbers. Use 4. to prove that
$$\frac{2ab}{a+b} \leq \sqrt{ab}$$
7. Let $a, b, x, y \in \mathbb{R}$. Prove that
$$(ab + xy)^2 \leq (a^2 + x^2)(b^2 + y^2)$$
8. Let a and b be real numbers. Use Triangle Inequality to prove that
$$||a| - |b|| \leq |a - b|.$$
9. Let $x, y \in \mathbb{R}$. Prove that
$$x > y - \varepsilon \quad \text{for all } \varepsilon > 0 \quad \text{if and only if} \quad x \geq y$$
10. Prove **Mathematical Induction (Theorem 1.2.2 page19)**.