Math SSRU

Assignment 2
MAC3309 Mathematical Analysis

Topic	Completeness Axiom & Functions	**Score**	10 marks
Time	2*nd* Week		
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1. Let $A = \left\{ \frac{n-1}{n+1} : n \in \mathbb{N} \right\}$. Find $\inf A$ and $\sup A$ with proving them.

2. Let $A = \left\{ \frac{1}{n^2+1} : n \in \mathbb{N} \right\}$. Find $\inf A$ and $\sup A$ with proving them.

3. Prove **Approximation Property for Infimum (API)**.
   If A has an infimum and $\varepsilon > 0$ is any positive number, then there is a point $a \in A$ such that
   $$\inf A \leq a < \inf A + \varepsilon.$$

4. Let r be a rational number and s be an irrational number. Prove that
   4.1 $r + s$ is an irrational number.
   4.2 if $r \neq 0$, then rs is always an irrational number.

5. Show that $\sqrt{2}$ is an irrational number.

6. Let $\sqrt{K} \in \mathbb{Q}^c$ and $a, b, x, y \in \mathbb{Z}$. Prove that
   $$\text{if } a + b\sqrt{K} = x + y\sqrt{K}, \text{ then } a = x \text{ and } b = y.$$

7. Prove **Theorem 1.3.13** : If x be a real number, then there exists an $n \in \mathbb{Z}$ such that
   $$n - 1 \leq x < n.$$

8. Use Theorem 1.3.13 to prove **Density of Rationals** :
   If $a, b \in \mathbb{R}$ satisfy $a < b$, then there is a rational number r such that
   $$a < r < b.$$

9. Use the Density of Rationals to Prove **Density of Irratioals** :
   If $a, b \in \mathbb{R}$ satisfy $a < b$, then there is an irrational number t such that
   $$a < t < b.$$

10. Let $f(x) = x^2 e^{x^2}$ where $x \in \mathbb{R}$. Show that f is 1-1 on $(0, \infty)$.