Math SSRU

Assignment 3
MAC3309 Mathematical Analysis

Topic	Limit of Sequences & Limit Theorems	**Score**	10 marks
Time	3*rd* Week		
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1. Use definition to prove that $\lim_{n\to\infty} \dfrac{2n+1}{n+1}$ exists.

2. Use definition to prove that $\lim_{n\to\infty} \dfrac{n^2}{n^2+1}$ exists.

3. Prove by contradiction to show that $\lim_{n\to\infty} \sin\left(\dfrac{n\pi}{2}\right)$ does not exist (DNE).

4. Assume that $x_n \to 1$ as $n \to \infty$. Show that

$$\frac{1}{(x_n)^2} \to 1 \text{ as } n \to \infty.$$

5. Assume that $x_n \to 0$ as $n \to \infty$. Show that

$$\frac{1+(x_n)^2}{x_n+1} \to 1 \text{ as } n \to \infty.$$

6. Let $\alpha \in \mathbb{R}$ and $\{x_n\}$ be a convergent sequence. Prove that

$$\lim_{n\to\infty} (\alpha x_n) = \alpha \lim_{n\to\infty} x_n.$$

7. If A has a finite infimum, then there is a sequence $x_n \in A$ such that

$$x_n \to \inf A \quad \text{as} \quad n \to \infty.$$

8. If $\{x_n\}$ is a convergent sequence, then

$$\lim_{n\to\infty} \frac{1}{x_n} = \frac{1}{\lim_{n\to\infty} x_n}$$

when $\lim_{n\to\infty} x_n \neq 0$ and $x_n \neq 0$.

9. Let $\{x_n\}$ be convergent such that converges to a. Prove that

$$\lim_{n\to\infty} |x_n| = |a|.$$

10. Let $x_n > 0$ such that converges to $a > 0$, then prove that

$$\lim_{n\to\infty} \sqrt{x_n} = \sqrt{a}.$$