Math SSRU

Assignment 5
MAC3309 Mathematical Analysis

Topic	Topology on $\mathbb{R}$ and Limit of functions	**Score**	10 marks
Time	$5th$ Week		
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1. Let $U \subseteq \mathbb{R}$ be a nonempty open set. Assume that U has a supremum and infimum. Show that

$$\sup U \notin U \text{ and } \inf U \notin U.$$

2. Prove **Theorem 3.3.11** : Let $A \subseteq \mathbb{R}$. Then $\bar{A}$ is closed.

3. Let A and B be subsets of $\mathbb{R}$. Prove that

$$(A \cup B)' = A' \cup B'.$$

   Use the resulte to confirm that $\overline{A \cup B} = \bar{A} \cup \bar{B}$.

4. Prove converse of **Theorem 3.3.13** :
   If the limit of every convergent sequence in F belongs to $F \subseteq \mathbb{R}$, then F is closed.

5. Use definition to prove that $\lim\limits_{x \to 1} x^2 + x + 1 = 3$.

6. Use definition to prove that $\lim\limits_{x \to -1} x^2 - x + 1 = 3$.

7. Use definition to prove that $\lim\limits_{x \to 0} \dfrac{x^2 + 1}{x + 1} = 1$.

8. Use definition to prove that $\lim\limits_{x \to 0} \dfrac{x^2 + 1}{x - 1} = -1$.

9. Let $y = f(x)$ be a real value function. Assume that

$$\lim_{x \to 1} \frac{f(x)}{x - 1} \quad \text{exists.}$$

   Prove that $\lim\limits_{x \to 1} f(x) = 0$.

10. Let $y = f(x)$ be a real value function. Assume that

$$\lim_{x \to 0} \frac{f(x)}{x} \quad \text{exists.}$$

    Prove that $\lim\limits_{x \to 0} f(x) = 0$.