Math SSRU

# Assignment 6
# MAC3309 Mathematical Analysis

**Topic** Limit Theorems of functions, One-sided Limits and Infinite Limits **Score** 10 marks
**Time** 6*th* Week
**Teacher** Assistant Professor Thanatyod Jampawai, Ph.D.
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1. Let
$$f(x) = \begin{cases} 2x+1 & \text{if } x > 1 \\ x+2 & \text{if } x \leq 1 \end{cases}.$$
Use definition to prove that $\lim\limits_{x\to 1} f(x)$ exists.

2. Use definition to prove that
$$\lim_{x\to\infty} \frac{x}{x-1} = 1.$$

3. Use definition to prove that
$$\lim_{x\to-\infty} \frac{x}{x-1} = 1.$$

4. Use definition to prove that
$$\lim_{x\to 1^+} \frac{1}{x-1} = +\infty.$$

5. Use definition to prove that
$$\lim_{x\to 1^-} \frac{1}{1-x} = +\infty.$$

6. Use definition to prove that
$$\lim_{x\to 2^+} \frac{1}{2-x} = -\infty.$$

7. Use definition to prove that
$$\lim_{x\to 2^-} \frac{1}{x-2} = -\infty.$$

8. Let $f$ and $g$ be real functions defined everywhere on $I$ except possibly at $a$ such that
$$f(x) \leq g(x) \quad \text{for all } x \in I - \{a\}.$$
Prove that if $f(x) \to \infty$ as $x \to a$, then $g(x) \to \infty$ as $x \to a$.