Math SSRU

Assignment 7
MAC3309 Mathematical Analysis

Topic Continuity & Uniform continuity **Score** 10 marks
Time 9*th* Week
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1. Use definition to prove that $f(x) = \dfrac{1}{x}$ is continuous at $x = 1$.

2. Prove that if f is continuous at a, then
$$\lim_{h\to 0} f(a+h) = f(a).$$

3. Prove that if $\lim\limits_{h\to 0} f(a+h) = f(a)$, then
$$f \text{ is continuous at } a.$$

4. Let E be a nonempty subset of $\mathbb{R}$ and $a \in E$. Suppose that $f : E \to \mathbb{R}$ is continuous at $a \in E$.
Prove that
$$\text{If } x_n \text{ converges to } a \text{ and } x_n \in E, \text{ then } f(x_n) \to f(a) \text{ as } n \to \infty.$$

5. Let $f(x) = x^2$. Prove that f is continuous on $\mathbb{R}$.

6. Use IVT to prove that $\ln x = 3 - 2x$ has **at least one real root** by using caculator to **find an interval** $[a, b]$ **of length 0.01** (the length of $[a, b]$ means $b - a$) that contain a root.

7. Show that
$$f(x) = x^2 - x$$
is uniformly continuous on $(0, 1)$.

8. Show that
$$f(x) = \frac{1}{1+x^2}$$
is uniformly continuous on $\mathbb{R}$.
(Hint: Use the fact that $(|x| - 1)^2 \geq 0$ for all $x \in \mathbb{R}$)

9. Let $f : I \to \mathbb{R}$ where I is open. Assume that f is continuous at a point $x_0 \in I$ and $f(x_0) > 0$.
Prove that there are positive numbers ε and δ such that
$$|x - x_0| < \delta \quad \text{implies} \quad f(x) > \varepsilon.$$

10. Let f and g be real functions. If f is differentiable at a and g is differentiable at $f(a)$, then $g \circ f$ is differentiable at a with
$$(g \circ f)'(a) = g'(f(a))f'(a).$$