



Assignment 8 MAC3309 Mathematical Analysis

Topic	Differentiability	Score	10 marks
Time	10th Week		
Teacher	Assistant Professor Thanatyod Jampawai, Ph.D. Division of Mathematics, Faculty of Education, Suan Sunandha Rajabhat University		

1. Show that $f(x) = x|x|$ is differentiable on \mathbb{R} .

2. Show that the function

$$f(x) = \begin{cases} x \sin\left(\frac{1}{x}\right) & : x \neq 0 \\ 0 & : x = 0 \end{cases}$$

is not differentiable at the origin.

Hint: Use the SCL to show that the limit does not exist.

3. Apply L'Hospital's Rule to find $\lim_{x \rightarrow \infty} x \left(\arctan x - \frac{\pi}{2} \right)$.

4. Use the Mean Value Theorem to prove that

$$\sin x \leq x \quad \text{for all } x \geq 0.$$

5. Use the Mean Value Theorem to prove that

$$\cos x - 1 \leq x \quad \text{for all } x \geq 0.$$

6. Find condition of $a \in \mathbb{R}$ satisfying

$$f(x) = ax^2 + 3x + 5$$

is strictly increasing on interval $(1, 2)$.

7. Let $f(x) = x^2 e^{x^2}$ where $x \in \mathbb{R}$.

7.1 Use IFT to show that f^{-1} exists and its differentiable on $(0, \infty)$.

7.2 Compute $(f^{-1})'(e)$.

8. Use the Inverse Function Theorem to prove that

$$(\arctan x)' = \frac{1}{1+x^2} \quad \text{for } x \in (-\infty, \infty).$$