



Assignment 9 MAC3309 Mathematical Analysis

Topic Reimann Integral **Score** 10 marks
Time 11th Week
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1. Let $f(x) = 1 - x^2$ on $[0, 1]$. Find $L(f, P)$ and $U(f, P)$ when $P = \left\{ \frac{j}{2^n} : j = 0, 1, 2, \dots, 2^n \right\}$
2. Let $f(x) = 3x^2$ on $[0, 1]$. Find $L(f, P)$ and $U(f, P)$ when $P = \left\{ \frac{j}{n} : j = 0, 1, 2, \dots, n \right\}$
3. Let $a > 0$ and $f(x) = ax^2 + 1$ where $x \in [-1, 1]$. Suppose that

$$U(f, P) - L(P, f) = 1 \quad \text{where} \quad P = \left\{ -1, -\frac{1}{2}, 0, \frac{1}{2}, 1 \right\}.$$

What is a ?

4. Let $f(x) = x^4$ where $x \in [0, 1]$. Find

$$U(f, P) - L(P, f)$$

in term of n when

$$P = \left\{ \frac{j}{n} : j = 0, 1, 2, \dots, n \right\}.$$

5. Let f be integrable on $[a, b]$ and $f(x) \geq 0$. Prove that

$$\int_a^b f(x) dx = 0 \quad \text{if and only if} \quad f(x) = 0 \text{ (zero function)}$$

6. Let

$$f(x) = \begin{cases} 1 & \text{if } 0 \leq x < 1 \\ 2 & \text{if } 1 \leq x < 2 \end{cases}$$

Show that f is integrable on $[0, 2]$

7. Let

$$f(x) = \begin{cases} 0 & \text{if } -1 \leq x < 0 \\ 1 & \text{if } 0 \leq x \leq 1 \end{cases}$$

Show that f is integrable on $[-1, 1]$

8. Let $n \in \mathbb{N}$ and define $f : [0, n] \rightarrow \mathbb{R}$ by

$$f(x) = \begin{cases} 1 & \text{if } 0 \leq x < 1 \\ 4 & \text{if } 1 \leq x < 2 \\ 9 & \text{if } 2 \leq x < 3 \\ \vdots & \vdots \\ n^2 & \text{if } (n-1) \leq x \leq n \end{cases}$$

If $\int_0^n f(x) dx = 385$, what is n .