# Solution Quiz 2 (8 a.m.)
# MAC3309 Mathematical Analysis

**Topic** Limit of Sequences **Score** 10 marks
**Time** 30 minutes (5*th* Week) **Semester** 2/2023
**Teacher** Assistant Professor Thanatyod Jampawai, Ph.D.
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1. **(5 marks)** Use the Definition to prove that

$$\lim_{n\to\infty} \frac{2n}{n+1} = 2.$$

***Proof.*** Let $\varepsilon > 0$. By Archimedean principle, there is an $N \in \mathbb{N}$ such that $\frac{1}{N} < \frac{\varepsilon}{2}$. Let $n \in \mathbb{N}$ such that $n \geq N$. We obtain $\frac{1}{n} \leq \frac{1}{N}$. Since $n+1 > n$, $\frac{1}{n+1} < \frac{1}{n}$. Hence,

$$\left|\frac{2n}{n+1} - 2\right| = \left|\frac{2n-2(n+1)}{n+1}\right| = \frac{2}{n+1} < \frac{2}{n} \leq \frac{2}{N} < \varepsilon.$$

Thus, $\lim_{n\to\infty} \frac{2n}{n+1} = \frac{1}{2}$. □

2. **(5 marks)** Use the Definition to prove that

$$\lim_{n\to\infty} \frac{2n^2}{n+1} = +\infty.$$

***Proof.*** Let $M \in \mathbb{R}$. By Arichimedean property, there is an $N \in \mathbb{N}$ such that

$$N > \frac{M+2}{2}.$$

It's equivalent to $2N - 2 > M$.
Let $n \in \mathbb{N}$ such that $n \geq N$. Then $2n - 2 > 2N - 2$. Since $0 > -2$, $2n^2 > 2n^2 - 2$. We obtain

$$\frac{2n^2}{n+1} > \frac{2n^2-2}{n+1} = \frac{2(n-1)(n+1)}{n+1} = 2n - 2 > 2N - 2 > M.$$

Hence, $\lim_{n\to\infty} \frac{2n^2}{n+1} = +\infty$. □

**Solution Quiz 2 (1 p.m.)**
**MAC3309 Mathematical Analysis**

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1. **(5 marks)** Use the Definition to prove that

$$\lim_{n\to\infty} \frac{2n}{n^2+1} = 0.$$

***Proof.*** Let $\varepsilon > 0$. By Archimedean principle, there is an $N \in \mathbb{N}$ such that $\frac{1}{N} < \frac{\varepsilon}{2}$.
Let $n \in \mathbb{N}$ such that $n \geq N$. We obtain $\frac{2}{n} \leq \frac{2}{N}$. Since $n^2+1 > n^2$, $\frac{1}{n^2+1} < \frac{1}{n^2}$. Hence,

$$\left| \frac{2n}{n^2+1} - 0 \right| = \frac{2n}{n^2+1} = \frac{2n}{n^2} < \frac{2}{n} \leq \frac{2}{N} < \varepsilon.$$

Thus, $\lim_{n\to\infty} \frac{2n}{n^2+1} = 0$. □

2. **(5 marks)** Use the Definition to prove that

$$\lim_{n\to\infty} \frac{1-n^2}{n} = -\infty.$$

***Proof.*** Let $M \in \mathbb{R}$. By Arichimedean property, there is an $N \in \mathbb{N}$ such that

$$N > 1 - M.$$

It's equivalent to $1 - N < M$.
Let $n \in \mathbb{N}$ such that $n \geq N$. Then $-n \leq -N$. So, $1 - n \leq 1 - N$
Since $1 \leq n$, $1 - n^2 \leq n - n^2$. We obtain

$$\frac{1-n^2}{n} \leq \frac{n-n^2}{n} = \frac{n(1-n)}{n} = 1 - n \leq 1 - N < M.$$

Hence, $\lim_{n\to\infty} \frac{1-n^2}{n} = -\infty$. □